Allouer le capital au temps : introduire la migration du risque de crédit pour mesurer les risques temporels

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Présentation en ligne à la Chaire PARI, le 13 mai 2026

Gestion du capital et tarification des risques

- Une gestion appropriée du capital constitue un enjeu majeur dans les secteurs bancaire et assurantiel
- Une gestion moderne de l'entreprise consiste généralement à allouer du capital à des lignes d'activité spécifiques (voire à certains risques majeurs individuels)
- L'objectif est d'optimiser le profil risque/rendement du portefeuille, le risque étant mesuré par le capital alloué
- La tarification des contrats sous-jacents doit tenir compte du capital alloué et de son coût dans la marge de risque
- Le coût du capital comprend le spread lié à la notation de crédit, ainsi qu'un spread supplémentaire reflétant la stratégie de l'entreprise
- Le facteur d'actualisation utilisé pour calculer la marge de risque dans la tarification correspond alors au profit cible que l'entreprise souhaite atteindre

Allocation du capital et temps

- Si le risque se développe sur un horizon de long terme, il existe actuellement deux manières d'allouer le capital pour un tel risque :
  1. les actuaires exigent que le capital soit maintenu pendant toute la période (stratégie extrêmement prudente et coûteuse)
  2. En revanche, les régulateurs n'exigent un capital de solvabilité que pour les paiements/sinistres venant à échéance au cours de l'année civile suivante, ce qui signifie que cette approche néglige les risques de long terme
- Cette divergence peut s'avérer coûteuse dans les deux approches, soit en raison de la charge supplémentaire liée à la détention de ce capital, soit à cause d'une sous-estimation du risque
- Cette étude[1] poursuit un double objectif plus large :
  1. développer un cadre permettant de quantifier les risques liés au temps
  2. évaluer des stratégies d'allocation du capital en fonction du temps jusqu'au règlement final, afin de gérer efficacement les activités à longue traîne sans freiner leur développement

[1] Reprint disponible sur : https://www.tandfonline.com/doi/full/10.1080/03461238.2025.2509974

Modelling Stochastically an Insurance Company

- The value of an insurance company can be expressed in terms of the economic value of its assets, A, and of its liabilities, L
- Their time evolution is expressed in terms of two discrete-time stochastic processes:

$$\mathbf{A} = (A_t,\, t = 0, 1, 2, \ldots) \text{ and } \mathbf{L} = (L_t,\, t = 0, 1, 2, \ldots)$$

  defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where time t is in years
- Let us denote by $\mathcal{F}_t$ the σ-algebra generated by all the information available at time t
- The value of the institution is then described by the stochastic process

$$\mathbf{Z} = (Z_t,\ t = 0, 1, 2, \ldots) \quad \text{defined as} \quad Z_t = A_t - L_t.$$

- We denote by $\Delta Z_{t+1} = -(Z_{t+1} - Z_t), t = 0, 1, 2, \ldots,$ the negative change in value of the company over a time horizon of 1 year (from year t to year $t+1$), which is the time span relevant for solvency regulations.

The Capital Requirement

- At the beginning of each year, the regulator will require the company to *hold sufficient capital*

$$\mathsf{SCR}_t := \rho(\Delta Z_{t+1}|\mathcal{F}_t),$$

to safeguard against adverse developments (losses) of both the asset and liability side *during that year*

- Here $\rho(\cdot)$ denotes an appropriate risk measure. Solvency II defines this Solvency Capital Requirements (SCR) as:

$$\rho(\Delta Z_{t+1}|\mathcal{F}_t) = \mathrm{VaR}_\alpha(\Delta Z_{t+1}|\mathcal{F}_t) = \inf\{x \in \mathbb{R} \mid \mathbb{P}(\Delta Z_{t+1} > x|\mathcal{F}_t) \leq 1 - \alpha\},$$

where $\alpha = 99.5\%$

- The analysis of this paper will heavily rely on (conditional) translation invariance and homogeneity of ρ, i.e.

$$\rho(aX + b|\mathcal{F}_t) = a\,\rho(X|\mathcal{F}_t) + b, \quad a > 0, b \in \mathbb{R},$$

which is fulfilled by VaR

Economic Valuation of Insurance Liabilities (1/2)

- Let us define X_t as the random variable (rv), representing the claim payment by the insurance company in year t ($t = 1, \ldots, n$)
- The required solvency capital at time t then amounts to

$$\mathsf{SCR}_t = \rho(\Delta Z_{t+1}|\mathcal{F}_t) = \rho(X_{t+1} + L_{t+1} - L_t|\mathcal{F}_t) = \rho(X_{t+1} + L_{t+1}|\mathcal{F}_t) - L_t,$$

  where we have used translation-invariance of the risk measure and assumed the value of the assets is constant for ease of notation since we are focused on the liability risk here
- This amount is made available by the capital provider for an (excess) expected return, η_t, on the provided capital between time t and $t+1$
- The actual return at time $t+1$ is $\mathsf{SCR}_t + (L_t - L_{t+1}) - X_{t+1}$, so that

$$L_t = \eta_t \, \mathsf{SCR}_t + \mathbb{E}(L_{t+1} + X_{t+1}|\mathcal{F}_t)$$

Using the expression for SCR_t, this leads to

$$L_t = \frac{1}{1+\eta_t}\,\mathbb{E}(X_{t+1} + L_{t+1}|\mathcal{F}_t) + \frac{\eta_t}{1+\eta_t}\,\rho(X_{t+1} + L_{t+1}|\mathcal{F}_t)$$

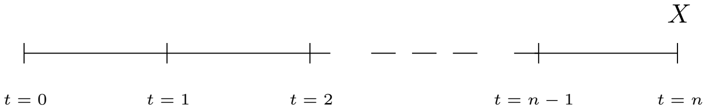
Economic Valuation of Insurance Liabilities (2/2)

- Let us assume that we know the distributions of all claim payments (and further cash-flows) X_t of future times $t = 1, \dots, n$ (conditional on the information $\mathcal{F}_{t-1}$ at the previous time point $t-1$)
- Let us further assume that the cost-of-capital rates η_t, $t = 0, \dots, n-1$, are known at the beginning as well
- This amount is made available by the capital provider for an (excess) expected return η_t on the provided capital between time t and $t+1$
- Then, the equation above can be used recursively (starting with $L_n = 0$) to determine the economic value of the liability L_0 at time $t = 0$
- Thus, L_0 is the amount to handle the run-off, including cost of capital
- A special case is the one-period model with a risk $X_1 = X$ at time $t = 1$. In this situation, with $L_1 = 0$, the equation simplifies to

$$L_0 = \frac{\mathbb{E}(X)}{1+\eta_0} + \frac{\eta_0}{1+\eta_0}\,\rho(X)$$

Capital Management Beyond One Step (1/2)

- Consider now how to value and manage an insurance liability X only due in the further future, at time $n > 1$, i.e. $X_n = X$ (and it is certain that it will not be faced earlier, like e.g. insuring a building that by contract will be completed by the end of Year $n-1$, or the launch of a satellite).
- Assume that the distribution of X is already known at $t = 0$, so that $\mathbb{E}(X|\mathcal{F}_{n-1}) = \mathbb{E}(X|\mathcal{F}_0) := \mathbb{E}(X)$ and $\mathbb{E}(\rho|\mathcal{F}_{n-1}) = \rho(X|\mathcal{F}_0) := \rho(X)$
- For simplicity, we treat it as a stand-alone event, so there are no additional claims from previous years to consider, and no new information emerges about it until the year it occurs



Facing a risk X at some future time n (no information in between)

Capital Management Beyond One Step (2/2)

- Beyond the regulatory requirements, the broader challenge here is to determine the appropriate amount of capital to allocate at each step until the ultimate point
- We propose to explore six strategies, starting with the one required by regulation:
  1. The coarse regulatory approach (ignoring credit migration risk)
  2. The consequent regulator's rationale (with credit migration risk)
  3. The conservative actuary (assigning the full capital at $t = 0$)
  4. The prudent actuary (building up linearly the capital until $t = t_{n-1}$)
  5. Buying a call option (for the capital at $t = t_n$)
  6. Buying credit rating protection

Determining the Cost of Capital over Time

- In our valuation methodology, we have introduced a cost of capital η_t that depends on the time t

- The question now is to determine this cost. There are two possibilities:
  1. The cost is fixed and does not depend on time. All one-year cost-of-capital rates η_t up to time $t = n - 1$ are known at $t = 0$ and $\eta_t = \eta_0$ (regulatory approach)
  2. The cost varies every year depending on the credit rating of the company and its target profit $\eta_t = \eta_t^{(k)}(s)$

- In the second case, $\eta_t^{(k)}(s)$ denotes the s-period cost-of-capital rate at time t for capital that will be held by the company (with credit rating k at time t) during the time period $[t, t+s]$

- We make the further assumption that $\eta_t^{(k)}(s)$ does not depend on the time t, so that solely the rating status k determines the value of $\eta_t^{(k)}(s) \equiv \eta^{(k)}(s)$ (for the same maturity and current rating status, the cost-of-capital rates are constant over time)

Deterministic Cost of Capital Rate (1/2)

- First, we examine the case where the cost of capital is known at $t = 0$ and fixed for the entire period
- This is the situation for the solvency requirement where the cost-of-capital is fixed at 6% (nowadays 4.75%)
- From our valuation equation, we get

$$L_{n-1} = \frac{1}{1+\eta_{n-1}}\,\mathbb{E}(X) + \frac{\eta_{n-1}}{1+\eta_{n-1}}\,\rho(X)$$

- The quantity L_{n-1} is then (already) deterministic at time $t = n-2$ (there is no randomness left), and we get, by applying the equation once more with $X_{n-1} = 0$ a.s. at $t = n-2$

$$\begin{aligned} L_{n-2} &= \frac{1}{1+\eta_{n-2}}\,\mathbb{E}(L_{n-1}|\mathcal{F}_{n-2}) + \frac{\eta_{n-2}}{1+\eta_{n-2}}\,\rho(L_{n-1}|\mathcal{F}_{n-2}) \\ &= \frac{L_{n-1}}{1+\eta_{n-2}} + \frac{\eta_{n-2}}{1+\eta_{n-2}}\,L_{n-1} = L_{n-1}, \end{aligned}$$

assuming that $\rho(c) = c$ for any constant c

Deterministic Cost of Capital Rate (2/2)

- In the same manner $L_0 = L_1 = \cdots = L_{n-1}$. Thus, if the risk-free interest rate is 0, at time $t = 0$ the premium (and reserve) is

$$L_0 = L_{n-1} = \rho(X) - \frac{1}{1 + \eta_{n-1}} \left(\rho(X) - \mathbb{E}(X)\right)$$

  needed for raising the additional capital $\text{SCR}_{n-1} = \rho(X - L_{n-1}) = \rho(X) - L_{n-1}$ at time $t = n - 1$
- The above could be interpreted as the regulator's view on how, at time $t = 0$, to deal with the risk X that will lead to possible claim payments between times $t = n - 1$ and $t = n$
- In this view, it would not be required to hold capital against this risk before $t = n - 1$, one only would have to set aside the capital cost $L_0 - \mathbb{E}(X)$ at $t = 0$ (needed later at $t = n - 1$)
- This approach tacitly assumes that the value of the cost-of-capital rate η_{n-1} is known at time $t = 0$ already

Introducing Credit Risk

- In practice, the insurer would fix the required cost of capital for pricing purposes every year based on the target profit of the company, which is related to its credit rating
- Thus, on one hand, an actuary may be concerned about the fact that this value of 6% can perhaps not be realized in the market, in particular when considering the future uncertainty about the credit rating of the insurance company and, on the other hand, risk management could be concerned to hold a risk on the books without capital associated to it
- It may therefore be more prudent to already allocate (parts of) the capital that will be needed at time $t = n - 1$ at earlier time points, when the rating (and resulting rate) is possibly more favorable
- This must be in a trade-off with the fact that holding capital for longer than the required year (starting at $t = n - 1$ in the above example) comes at an additional cost
- Clearly, η_0 is known at time $t = 0$, but all the future cost-of-capital rates $\eta_1, \ldots \eta_{n-1}$ are unknown and it makes sense to model them as random variables, which we will do in the sequel

Random Cost of Capital (1/3)

- Consider now the situation where in every single year we want to fulfill the regulatory requirement of future obligations and raise the necessary solvency capital according to the then available one-year cost of capital (according to our respective credit rating) until the end
- The resulting L_0 then represents the value of future liabilities in the situation when (in each year up to expiry) investors demand a spread for one-year time borrowing and renewing of capital to the next needed amount, depending on the then applicable credit rating of the company
- To simplify notation, define for any $i = 0, \dots, n-1$

$$\delta(\kappa_i) := \frac{1}{1 + \eta^{(\kappa_i)}(1)} < 1 \quad \text{and} \quad K(X) := \rho(X) - \mathbb{E}(X).$$

The valuation at $t = n-1$ then reads

$$L_{n-1}(\kappa_{n-1}) = \mathbb{E}(X) + (1 - \delta(\kappa_{n-1})) \cdot K(X) = \rho(X) - \delta(\kappa_{n-1}) \cdot K(X),$$

where $\delta(\kappa_{n-1})$ is an $\mathcal{F}_{n-1}$-measurable random variable, i.e. it is only known at time $t = n-1$ (when the credit rating at that point in time is known)

Random Cost of Capital (2/3)

- Under the assumption, $X_1 = \ldots = X_{n-1} = 0$, one can rewrite the value for any earlier times $t = 0, \ldots, n-2$ as

$$L_t(\kappa_t) = \delta(\kappa_t) \cdot \mathbb{E}(L_{t+1}|\mathcal{F}_t) + (1 - \delta(\kappa_t)) \cdot \rho(L_{t+1}|\mathcal{F}_t)$$

- Note that according to this representation the value of the future liabilities at time t can be interpreted as a convex combination of its expected value at time $t+1$ and its risk measure at time $t+1$, with the weights being given by the current one-year 'discount rate' due to the cost of capital
- A little calculation, using the translation-invariance and the homogeneity of the risk measure ρ, now shows that the value applied at $t = n-2$ gives

$$L_{n-2}(\kappa_{n-2}) = \rho(X) + f_{n-2}(\kappa_{n-2}) \cdot K(X),$$

where

$$f_{n-2}(\kappa_{n-2}) := \delta(\kappa_{n-2}) \cdot \mathbb{E}(-\delta(\kappa_{n-1})|\mathcal{F}_{n-2}) + (1 - \delta(\kappa_{n-2})) \cdot \rho(-\delta(\kappa_{n-1})|\mathcal{F}_{n-2})$$

is (solely) a function of the credit rating κ_{n-2} of the insurance company at time $t = n-2$

Random Cost of Capital (3/3)

- Iterative application of the same principle then gives for any $t = 0, \ldots, n-2$

$$L_t(\kappa_t) = \rho(X) + f_t(\kappa_t) \cdot K(X),$$

where

$$f_t(\kappa_t) = \delta(\kappa_t) \cdot \mathbb{E}(f_{t+1}(\kappa_{t+1})|\mathcal{F}_t) + (1 - \delta(\kappa_t)) \cdot \rho(f_{t+1}(\kappa_{t+1})|\mathcal{F}_t)$$

Recursively, the value of the future liabilities as a function of the credit rating κ_0 of the company at time $t = 0$ is

$$L_0(\kappa_0) = \rho(X) + f_0(\kappa_0) \cdot K(X)$$

- Instead of stochastics in stochastics for multi-period models, we end up with an analytical equation that can be determined recursively

Computation of the Spread from Default Probabilities

- start by estimating actual transition probabilities for the Markov chain model from market data
- Denzler *et al* 2006 have developed a model to link credit spread to default probability which fits market data very well by including fat-tailed jumps

$$s_{j,i} = (1 + \bar{Y}_{j,i}) \left[\frac{1}{[R + (1-R)(1-q_{j,i})]^{1/j}} - 1 \right]$$

$s_{j,i}$ is the (annual) spread for maturity j (in years) at time i, R is the recovery rate, $\bar{Y}_{j,i}$ is the default free yield and $q_{j,i}$ is the default probability up to maturity j at time i

- While among all financial institutions a recovery of 40% is considered typical, for insurance companies a recovery rate of 60% ($R = 0.6$) seems more appropriate
- Recall that we assume the spread depends only on the credit rating k at a given time point i, rather than on time i directly, which allows us to omit the index i
- The annual spread for remaining maturity j with current credit rating k is thus given by

$$s_j^{(k)} = \frac{1}{[0.6 + 0.4(1 - q_j^{(k)})]^{1/j}} - 1,$$

where $q_j^{(k)}$ is the probability to default within the next j years, given a current credit rating of k

Transition Probabilities for the Markov Chain Migration Model

- To compute the probability to default, we use our Markov chain assumption for the annual development of the rating
- For the one-year transition matrix, we choose a recent study of the rating agency S&P (Nick W Kraemer, S&P (2021), Table 21) that extends over 40 years from 1981 to 2020

S&P Global Corporate Average Transition Rates (1981-2020) for 1 year in percentage

From/to	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC/C**	**D**
AAA	89.85	9.35	0.55	0.05	0.11	0.03	0.05	0
AA	0.50	90.77	8.08	0.49	0.05	0.06	0.02	0.02
A	0.03	1.67	92.61	5.23	0.27	0.12	0.02	0.05
BBB	0	0.10	3.45	91.93	3.78	0.46	0.11	0.17
BB	0.01	0.03	0.12	5.03	85.99	7.51	0.61	0.70
B	0	0.02	0.08	0.17	5.18	85.08	5.66	3.81
CCC/C	0	0	0.12	0.20	0.65	14.72	50.90	33.41

Cost of Capital as a Function of Maturity

- The over-all cost of capital $\eta^{(k)}(j)$ for a maturity of j years is then given by

$$\eta^{(k)}(j) = \sum_{i=1}^{j} (s_i^{(k)} + \eta_r).$$

That is, in each year i the regulatory cost of capital for one year, η_r, is augmented by the annual credit spread $s_i^{(k)}$ for the remaining maturity i at that point

Cost of capital $\eta^{(k)}(j)$ as a function of maturity j and current credit rating state k

	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$
AAA	0.0600	0.1200	0.1801	0.2402	0.3003
AA	0.0601	0.1202	0.1803	0.2405	0.3008
A	0.0602	0.1205	0.1807	0.2411	0.3014
BBB	0.0607	0.1215	0.1824	0.2435	0.3048
BB	0.0628	0.1264	0.1908	0.2558	0.3214
B	0.0755	0.1539	0.2339	0.3146	0.3956
CCC/C	0.2143	0.3951	0.5522	0.6925	0.8206

A Pareto Risk With a Time Horizon of 5 Years

- To assess the different strategies, it is necessary to choose a distribution for the risk X and a time horizon
- Let us now assume that the risk X is Pareto distributed with distribution function $F_X(x) = 1 - (x/x_0)^{-1/\xi}$, $x > x_0$
- We choose the parameters to be $x_0 = 1$ and $1/\xi = 1.8$, which is fat tailed with a non-converging second moment
- We furthermore choose $n = 5$ years and the risk measure ρ to be the Value-at-Risk at safety level $\alpha = 99.5\%$
- This leads to $\mathbb{E}(X) = x_0/(1-\xi) = \mathbf{2.25}$ and $\rho_\alpha(X) = x_0/(1-\alpha)^\xi = \mathbf{18.9824}$
- Under Strategy 1 (where a constant $\eta_r = \eta_4 = 0.06$ is assumed and applied), we obtain the benchmark value

$$L_0^{(1)} = \mathbb{E}(X) + 0.0566\, K(X) = \mathbf{3.19705}$$

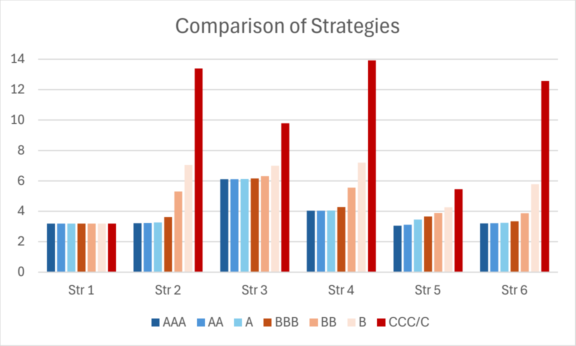
- This represents the liability value required by the regulators, but neglecting the risk of credit migration as we have seen earlier

Capital Allocation to Time: Six Strategies

We explore six ways of solving the problem of capital allocation to time:

1. The coarse regulatory approach that allocates capital only at $t = t_{n-1}$ and fixes the cost of capital at 6% neglecting the credit migration risk
2. The consistent regulatory rationale that takes into account credit migration risk (stochastic cost of capital), but keeps allocating the capital only at $t = t_{n-1}$
3. The conservative actuarial approach that allocates capital at time $t = t_0$ and keeps it all the time, with the stochastic cost of capital
4. The prudent actuary approach that allocates dynamically capital so to reach the full capital at $t = t_{n-1}$, with a fixed cost of capital
5. Buying a (capped) call option on the excess of the eventual loss over the expected value of risk X (the excess needed is bounded from above by $\rho(X) - \mathbb{E}(X)$), so it is in fact called a bull spread in financial terms
6. Buying a credit protection: One can lock in the liability $L_0^{(1)}$ of Strategy 1 (which the regulator requires), and – instead of any other measures – buy a credit derivative, which protects the company against any adverse future developments

Allocating Capital to Time



Resulting liability values for each strategy and each initial credit rating

- Strategy 2 is the minimum risk that is realistic and we compare to this one
- We observe that Strategy 5 is quite attractive if such a call option is available
- Strategy 6 can be also very attractive
- For low rating, strategy 4 can also be attractive

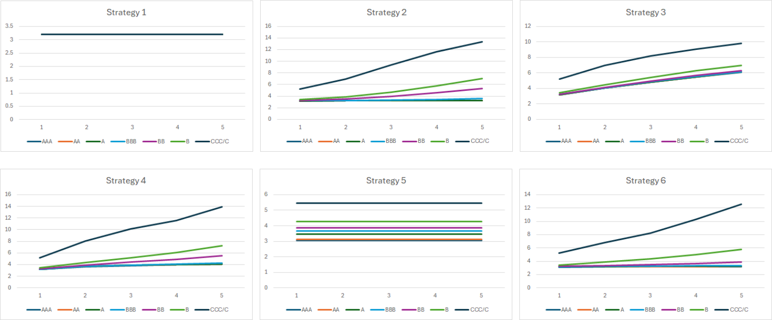
Valuation for All Strategies

Valuation results for all the strategies as shown in the graph
(Strategy 2 is the benchmark in red. In green better strategies)

	Str 1	**Str 2**	**Str 3**	**Str 4**	**Str 5**	**Str 6**
AAA	3.19705	*3.2150*	6.1147	4.0342	3.0521	3.2108
AA	3.19705	*3.2344*	6.1178	4.0399	3.1206	3.2186
A	3.19705	*3.2623*	6.1256	4.0576	3.4564	3.2427
BBB	3.19705	*3.6276*	6.1583	4.2773	3.6550	3.3475
BB	3.19705	*5.3078*	6.3199	5.5598	3.8815	3.8739
B	3.19705	*7.0463*	6.9926	7.1952	4.2663	5.7787
CCC/C	3.19705	*13.3924*	9.7918	13.9259	5.4504	12.5692

Sensitivity with Respect to Time Horizon

We explore how valuation changes with the time horizon n over which the liability materializes and terminates



- Strategies 2, 4, and 6 have the strongest sensitivity to the time horizon as one would expect
- As for Strategy 1, in the absence of discounting the values for Strategy 5 do not depend on the choice of n
- Strategy 5 immunizes against the time variation (without discounting)
- Note that for $n = 1$ the values of Strategies 2, 4 and 6 coincide
- The variation of the valuation with the time horizon is smooth but non-linear
- The sensitivity with respect to the time horizon is particularly pronounced for credit ratings BB or worse

Compensating for Random Cost of Capital Rates

- From a regulatory perspective, it is instructive to see how the fixed cost-of-capital rate η_r in Strategy 1 must be *adjusted* to account *for real rate fluctuations* tied to credit states
- This question is of interest for the 'consequent regulator' described in *Strategy 2*
- For credit states *BBB and below*, using $\eta_r = 6\%$ in Strategy 1 becomes *problematic*, especially for longer horizons n

Implied deterministic values η_r^* for Strategy 2 as a function of time horizon n (rounded to the fourth digit)

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
AAA	6.00%	6.01%	6.03%	6.06%	6.12%
AA	6.01%	6.02%	6.05%	6.12%	6.25%
A	6.02%	6.04%	6.10%	6.21%	6.44%
BBB	6.07%	6.21%	6.65%	7.53%	8.97%
BB	6.28%	8.05%	11.31%	16.07%	22.36%
B	7.55%	10.94%	17.31%	26.96%	40.18%
CCC/C	21.43%	38.61%	73.41%	126.41%	199.33%

Conclusion (1/2)

- Gérer efficacement la dimension temporelle dans l'estimation et la gestion des risques demeure l'un des défis fondamentaux des mathématiques actuarielles et de la gestion des risques
- Nous développons ici un cadre permettant d'évaluer l'allocation du capital au temps, en nous appuyant sur une méthodologie de valorisation des engagements afin d'analyser différentes stratégies d'allocation du capital
- L'avantage de ce cadre est qu'il traite le problème multi-étapes de manière analytique, sans avoir recours à des processus stochastiques imbriqués
- Nous montrons que l'approche recommandée par le régulateur néglige à la fois le risque de crédit lié au coût du capital et les principes fondamentaux d'une gestion prudente des risques
- Allouer l'intégralité du capital au début du contrat rend l'engagement excessivement coûteux, dépassant les exigences réglementaires de plus de 90% dans tous les cas
- Une constitution progressive du capital apparaît comme un compromis, réduisant le coût des engagements tout en exigeant un certain niveau de capital dès le début du contrat

Conclusion (2/2)

- Nous examinons également le transfert de l'ensemble du risque des engagements vers une autre institution, ainsi que la couverture du risque de migration de crédit
- Bien que le transfert de risque apparaisse comme une solution coût-efficace, des difficultés pratiques subsistent en raison des contraintes réglementaires et des réticences du marché
- La couverture du risque de migration de crédit (Stratégie 6) apparaît, dans le cadre de nos hypothèses, comme une alternative viable à la Stratégie 2. Toutefois, elle ne résout pas le problème de gestion des risques lié à l'absence d'allocation de capital au début du contrat
- Une piste de recherche future consisterait à formaliser la manière dont des stratégies prudentes d'allocation du capital pourraient améliorer les notations de crédit, en étendant le modèle actuel
- Cela impliquerait de passer d'un cadre exogène à un cadre endogène, dans lequel l'application uniforme d'une stratégie de capital à l'ensemble des portefeuilles influencerait la notation globale de l'entreprise

APPENDICES

Strategy 1: The Coarse Regulatory Approach

- As a benchmark, we start with the regulatory approach to assume the cost-of-capital rate in the future is known at time $t = 0$ and equal to $\eta_{n-1} = \eta_r = 0.06$

$$L_0^{(1)} = \mathbb{E}(X) + \frac{\eta_r}{1+\eta_r}\left(\rho(X) - \mathbb{E}(X)\right) = \mathbb{E}(X) + 0.0566 \cdot K(X)$$

- This approach neglects the fact that modern risk management the actuaries would use a value higher than 6% for the cost-of-capital, taking into account the credit spread and the profit target
- Even in cases where regulators do not mandate capital reserves for this risk at time $t = 0$, it could be risky from a risk management perspective to maintain such a risk on the books without assigned capital
- There is a need to explore other ways of allocating capital to long-tail business

Strategy 2: The Consequent Regulator's Rationale

- Following the regulator's approach, the company will not allocate solvency capital for dealing with X before time $t = n-1$ and, at that point, will raise SCR_{n-1} for the last year
- In presence of credit rating risk, extra capital is needed to protect against adverse developments of the credit rating and correspondingly reserves need to be sufficient to afford the potentially higher costs of raising SCR_{n-1} at $t = n-1$
- We also take into account the possibility of a costly bankruptcy
- Since $L_n = 0$ and $X_n = X$, the value for the liability, for each credit state κ_{n-1} at time $t = n-1$, becomes

$$\begin{aligned} L^{(2)}_{n-1}(\kappa_{n-1}) &= \delta(\kappa_{n-1})\,\mathbb{E}(X) + (1-\delta(\kappa_{n-1}))\,\rho(X) \\ &= \rho(X) - \delta(\kappa_{n-1})\,(\rho(X) - \mathbb{E}(X)) \end{aligned}$$

- Under the rather natural assumption

$$L^{(2)}_{t+1}(1) < L^{(2)}_{t+1}(2) < \ldots < L^{(2)}_{t+1}(7) < \eta^{(7)}(n-t-1)\cdot\rho(X) + L^{(2)}_{t}(\kappa_t)(1-\eta^{(7)}_t(n-t-1))$$

  for all κ_t (which will be empirically checked in the calculations later on), the quantity $\rho(Y_{t+1}|\kappa_t)$ can easily be determined for each credit state κ_t
- Starting with $L^{(2)}_{n-1}(\kappa_{n-1})$ from (6.1), the implicit equation for the reserves then leads to a simple way of calculating $L^{(2)}_{n-2}(\kappa_{n-2})$ for each credit state κ_{n-2}
- This recursive procedure can be applied for all smaller values of t as well, eventually arriving at $L^{(2)}_0(\kappa_0)$ for any initial credit state κ_0

Strategy 3: The Conservative Actuary

- The conservative actuary prefers to allocate the capital needed at time $t = n - 1$ already at time $t = 0$, and hold it throughout the entire time period $[0, n]$
- The conservative actuary would argue that since the risk is on the book, the capital to support it should also be on it
- The cost for this at time $t = 0$ is

$$\eta^{(\kappa_0)}(n) \cdot (\rho(X) - L_0^{(3)}),$$

- which leads to

$$L_0^{(3)}(\kappa_0) = \mathbb{E}(X) + \frac{\eta^{(\kappa_0)}(n)}{1 + \eta^{(\kappa_0)}(n)}(\rho(X) - \mathbb{E}(X)).$$

  where κ_0 is the initial credit state
- This will typically be considerably larger than the solvency benchmark
- The longer the time until ultimate, the more penalizing this strategy becomes, as it ties up capital for the entire period

Strategy 4: The Prudent Actuary

- The prudent actuary will allocate every year already some part of the eventually needed capital until the time $t = n - 1$ when it is finally required by the regulator
- In this way, the credit migration risk is improved in a cheaper way than under Strategy 3, while still maintaining an initial allocation of capital
- mathematically this leads to a considerably more complex procedure than for the previous strategies: SCR_{n-1} now not only depends on κ_{n-1}, but also on all other earlier credit states $\kappa_0, .., \kappa_{n-2}$, since the capital costs at $t = n - 1$ depend on all the previously visited credit states
- To simplify the analysis, it is somewhat natural to assume that we allocate the same capital level $C_0 = \ldots = C_{n-2} = C(\kappa_0)$ for all $t = 0, \ldots, n - 2$ (at time $n - 1$ we then raise the actually needed remaining amount)
- Its value is determined at the beginning according to the credit state κ_0. An intuitive choice for raising a constant amount is

$$C(\kappa_0) = (\rho(X) - L_0^{(2)}(\kappa_0))/n,$$

where $L_0^{(2)}(\kappa_0)$ is the initial liability value under Strategy 2

Strategy 5: Buying a Call Option

- An alternative is to buy a (capped) call option on the excess of the eventual loss over the expected value of risk X (the excess needed is bounded from above by $\rho(X) - \mathbb{E}(X)$, so it is in fact a bull spread in financial terms
- The price of this call option together with $\mathbb{E}(X)$ then constitutes the initial liability value

$$L_0^{(5)} = \mathbb{E}(X) + \mathbb{E}_{\mathbb{Q}}((\min(X, \rho(X)) - \mathbb{E}(X))_+).$$

  where $\mathbb{E}_{\mathbb{Q}}$ is the risk neutral expectation
- Since a respective market is clearly not liquid, it will not be realistic to infer the pricing measure $\mathbb{Q}$ from a market
- Here, we use the simple pricing kernel

$$\mathbb{E}_{\mathbb{Q}}(\cdot) = (1 + \theta^{(5)})\mathbb{E}_{\mathbb{P}}(\cdot)$$

  for some $\theta^{(5)} > 0$, where $\mathbb{P}$ is the physical measure (other choices of $\mathbb{Q}$ are of course easily possible)
- One could also interpret the call option as a reinsurance cover with a bounded layer above a retention $\mathbb{E}(X)$, priced with an expected value principle with relative safety loading θ according to an expected value premium principle

Strategy 6: Buying Credit Rating Protection

- Another alternative is to manage the risk of a worsening of the credit state of the company is the following. One can lock in the liability $L_0^{(1)}$ of Strategy 1 (which the regulator requires), and – instead of any other measures – buy a credit derivative, which protects the company against any adverse future developments (i.e., against a bad credit state at time $n-1$ when the allocation of the capital is due)
- The drawback of this technique is the lack of initial capital allocation. This creates the wrong incentives for underwriters, as they are not required to generate a return on risk capital in the first year
- In addition, one then also needs to add protection against the costs of bankruptcy which might occur at any point in time up to $t = n-1$. This results in a financial product whose payoff at time $n-1$ is contingent on the credit state of the company at that time and of the form

$$W(\kappa_0) = \begin{cases} s_1^{(j)} \cdot (\rho(X) - L_0^{(1)}) & \text{with prob. } \hat{p}_{\kappa_0,j}^{(n-1)} \text{ for } j = 1, \ldots, 7. \\ \eta^{(7)}(n-t) \cdot \rho(X - L_0^{(1)}) & \text{with prob. } \hat{p}_{\kappa_0,D}^{(t)}. \end{cases}$$

- For simplicity of exposition, in this paper we choose to price this credit derivative using again $\mathbb{E}_{\mathbb{Q}}$ and the same pricing kernel as for the call option under Strategy 5, i.e.

$$L_0^{(6)} = L_0^{(1)} + \mathbb{E}_{\mathbb{Q}}(W(\kappa_0)) = L_0^{(1)} + (1+\theta^{(6)})\,\mathbb{E}_{\mathbb{P}}(W(\kappa_0)),$$

  where in comparison to Strategy 5 a lower value for $\theta^{(6)}$ may be realistic here, as the market for credit derivatives is quite liquid