Risk and statistics: the pricing and prudential regulations fallacy

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Abstract:

A prudential regulation such as Solvency II, as well as some positions in the debate over fair pricing regulation, rely on an intellectual misunderstanding and the consequent misuse of statistics to apprehend risks. Indeed, the uniqueness of the mathematical formalism that enables description of heterogeneity and randomness has led us to mistakenly transpose insights from one domain onto another, distorting our intuitions and damaging our understanding of risk. We distinguish between the two types of phenomenon and draw general conclusions on the scope of relevance of statistics to apprehend risks. Then, we apply this framework on two major regulatory debates: (1) should fair price be defined and enforced taking the cost of risk as the framework of reference? And (2) should capital requirements rely on statistical measures or on another approach?

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"Not only is it not right, it isn't even wrong."

W. Pauli

Introduction

For more than two centuries, insurers and their actuaries successfully used statistics to run their business. As a consequence, a belief emerged: statistics and actuarial tools would be apposite to manage risk and make decision when facing alea. This belief has become widespread in the last decades, tending to be implemented in various types of regulations. In the USA, for example, pure premium (that is, technical expectancy) tends to be considered as a sound way to capture the *individual* policyholder exposure and interest, and therefore to be judged as the frame of reference to assess the fairness of a price. In the European Union, the Solvency II VaR-based capital requirements have been considered cutting edge regulation to improve prudential supervision and ERM. We believe that it is not apposite to do so.

Such regulations are based on an underlying confusion between the tools used for managing risk and those used for steering business profitability. On the one hand, insurers, like all companies, individuals or governments, are subject to uncertainties due to the fact of their very existence and activity. These uncertainties may be adverse, and placing them under control is a question of risk management. On the other hand, the activity of insurers is to take on the risks of others. This is not a question of managing risks *per se*, but of risk pooling – of managing heterogeneity (of outcomes): an insurer will make use of the fact that some of its clients have accidents while others do not. The business model of a financial institution thus rests not on the management of risks but on the management of heterogeneity. The difference is crucial: describing heterogeneity is a predictive matter whereas describing risk, by definition, is not a matter of prediction. As a consequence, we have to wonder whether "risk models" and related statistics, efficient for pricing and thus managing *heterogeneity*, are appropriate tools or not for *risk* management.

To put it another way, such practices are based on the conflation between the points of view of different agents of the risk industry. When signing an insurance contract, for example, the policyholder and the insurer are faced with the same underlying phenomenon – an unknown regarding whether or not this particular contract will result in compensation – yet the parties are not in the same *situation* in respect to it: the policyholder, even if he has a certain knowledge of statistics, *does not know* what will happen to himself, whereas the insurer *knows* that, statistically, across all its policyholders, he will pay x amount. However, while these two actors are in different situations, we are used to having recourse to the same formalism to describe the common phenomenon with which they are faced when signing the contract, *regardless of their viewpoint*: The distribution function and the statistical indicators such as expectancy or VaR describe both the random phenomenon (here, the outcome seen by the policyholder) and the heterogeneous population (here, the outcomes seen by the insurer). Is this valid?

The answer given to us by physics is: potentially no. Indeed, physics tends to adapt its choice of model to the observer's position: ice observed over a short period of time, for example, to build an ice hotel, will require equations from solid mechanics, whereas the same ice observed over a long period of time, such as for analyzing the flow of a glacier tongue, will require equations from liquid mechanics. By contrast, financial regulations tend to use a single analytic framework,, thus disregarding the decision-maker's situation. This can lead to transposing tools for situations where they are relevant onto situations where they are not. It happens for example, when one transposes the insurer financial equilibrium solutions and pricing models on the policyholder fairness issue. Or, regarding prudential questions, when one transposes portfolio assessment solutions on risk management issues. These tools would thus serve as a vehicle for a cognitive representation of phenomena that is not adapted to a correct understanding of the situation, resulting in a weakened quality of regulation and decision-making.

Inspired by Doherty (2013) idea of endogenizing the observer, this paper positions itself at the level of interaction between the phenomenon and the observer/decision-maker, to identify the situations in which the usual tools and concepts are not relevant for understanding and regulating risks, as opposed to those situations in which they are. The first section will set-up the conceptual framework and accordingly characterize the field of relevance of the above-mentioned statistical indicators (and associated estimators), as well as the consequences of their use outside of this field ; the reader eager to focus on operational applications can go straightforwardly to the summary ending this part. The second section will focus on its application on pricing regulation. Finally, the third section will deal with its application on prudential regulations. Both second and third sections will be organized as follows: first, summarizing the regulatory debate in major countries; then presenting our critical analysis; finally proposing solutions.

1. Underlying conceptual framework and derived general results

1.1 Definitions

We focus on characterizing the intersection of an underlying random phenomenon and the decision-maker who seeks to understand it. We define this context in a generic way with the word *situation*:

Definition 1: A situation is a context where a future phenomenon and an agent come together, *such that:*

(i) the phenomenon will result in one or more realizations in the eyes of the agent,

(ii) *before each realization, the agent does not know what will result from the realization,*

(iii) *the dispersion of the realizations of which the agent can conceive is significant in regard to his decision-making threshold.*

For example, taking out an insurance policy relates to a unique underlying phenomenon (whether the policyholder will develop a cancer). The dispersion of the individual outcome is significant, as one's would behave in a different way if he knew the outcome before signing the contract. Notwithstanding the uniqueness of the phenomenon, the policyholder's and the insurer's situations are not identical, as the policyholder signs a single contract and observes a single realization, whereas the insurer signs a large number of contracts.

The insurer's situation differs *operationally* from that of the policyholder in that as he signs a large number of contracts, the underlying phenomenon from his perspective shall be grasped outside of one isolated case. Taken as a whole, the insurer's portfolio will have a deterministic result thanks to the law of large numbers. The observer can *forecast*[2]*.* Thus, there is a difference in the *nature* of the observer's situation depending on whether the underlying phenomenon repeats itself or not: in one case, the situation remains random, whereas in the other it is almost deterministic. We can therefore identify two types of situations:

Definition 2: Theoretically, in an idealistic perspective, random situations *are situations where the observer will only observe a single realization;* heterogeneous situations *are situations where the observer will observe an infinity of realizations.*

Operationally, in a pragmatic perspective, we are in a random situation when the number of realizations for the observer in question is "low" and in a heterogeneous situation when the number is "high". The transition area between "low" and "high" is defined by the consequences held by its crossing: when the eventuality of a significant deviation from an expectation is

[2] With a certain margin of error, of course, as in any forecast, be it in physics.

negligible in the eyes of the decision-maker (i.e. deterministic context), it is a heterogeneous situation. *In the opposite case (remaining uncertainty non negligible), it is a* random situation.

Let us take two examples in an insurance company: the actuary and the Chief Investment Officer. The actuary, who prices and sells thousands or millions of similar policies, each of which resulting or not in a compensation, does so in a *heterogeneous situation*. Conversely, the CIO chooses the strategic asset allocation for a period that is significant in relation to the duration of his tenure, not knowing what will be the overall return of each asset class: he will make this kind of decision very few times during his tenure, and as such he is in a *random situation*.

In the definition we propose, three points are to be highlighted:

(i) The definition of the type of a situation is totally *contingent upon the observer*. Therefore, this concept cannot be assimilated into the distinction between insurable risk and uninsurable risk, which is an intrinsic feature of the event, indifferent to the decision-maker. The purpose of our analysis is precisely to draw conclusions in terms of the relevance of descriptive tools applicable to the phenomenon *as regards the agent.*

(ii) The second point is that the distinction between a random situation and a heterogeneous situation is based on the number of *future* realizations of the phenomenon: the precise knowledge of the past does not matter *a priori* in the distinction that we make here between random situations and heterogeneous situations. As a corollary, the distinction between risk and uncertainty (Knight, 1921) is not fundamental in our view. However, one could underscore of the *de facto* frequent proximity of random situations and uncertain situations.

(iii) Lastly, in our view, as regards the pertinence of statistical indicators to the basis of a reasoning, the often highlighted opposition between thin-tailed and fat-tailed distributions (e.g. Taleb, 2007) is secondary to the distinction between randomness and heterogeneity. Of course, this affects the number of realizations that mark the threshold between a random situation and a heterogeneous situation, but as we will see, the difference of analysis required to manage these situations has to do with the difference in the nature of these situations and not with the thickness of the distribution tail.

1.2 Graphic intuition

The classic mathematical representations of a random phenomenon according to the observer's situation are as follows:

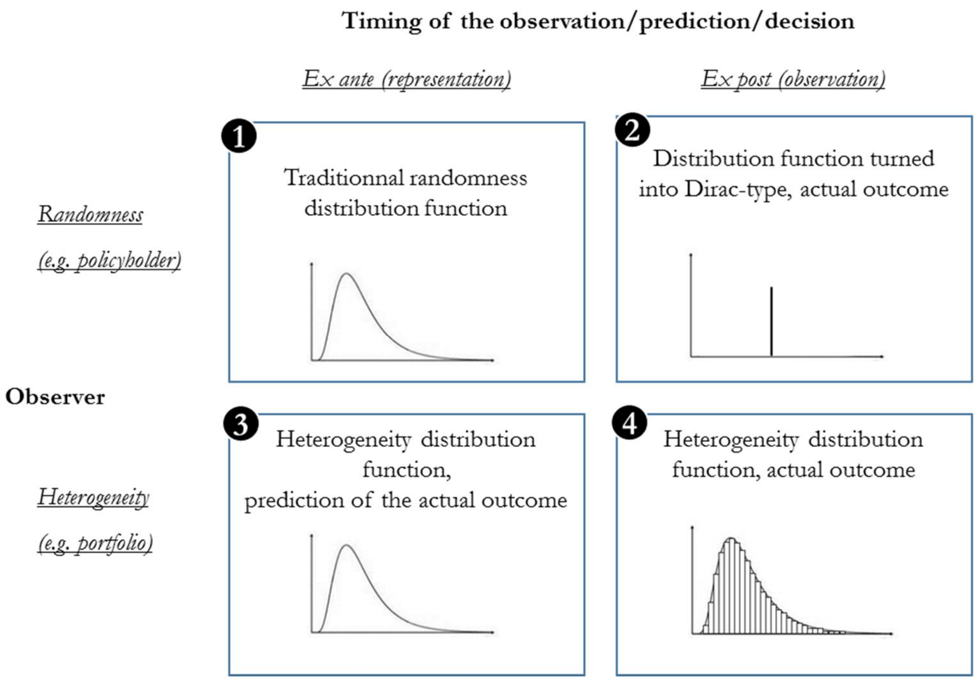

Chart 1: distribution functions according to situation

In figure 1, the first row corresponds to an observer placed in a random situation (e.g. a policyholder subscribing annuity), and the second to an observer in a heterogeneous situation (an insurer considering its large portfolio). The first column precedes the realization of the phenomenon (while signing the contract); the second column corresponds to the view after the realization of the random event(s) (after the death of the policyholder).

Function 2 is a Dirac delta function, as the policyholder died a given number of years after signing the contract. Function 4 is, say, a log-normal function, representing the life length actually observed on the whole set of policyholders. Both 2 and 4 represent an *actual* outcome.

Function 3 is also a log-normal function. Due to the law of large numbers, it properly characterizes the future outcome: it is a deterministic description of the result with which our observer will be faced. In contrast, the distribution function of 1 does not correspond to a description of an actual outcome.

The difference in nature between a random situation and a heterogeneous situation appears clearly in the way that the phenomenon's distribution function, in the case of heterogeneity, remains stable in 3 and 4, whereas in the case of randomness it transforms from 1 to 2. In other words, 3 is a prediction, whereas 1 is not:: the statistical quantities associated with 3 are estimators of the dispersion that will actually be observed, which is not true of 1.

Since *ex post* functions 2 and 4 are different after the phenomenon's realization, one might naïvely wonder whether it would be justifiable to assimilate *ex ante* functions 1 and 3, neglecting the observer's situation. This is precisely the questions we ask: (1) mathematically, is function 1 an apt representation of the phenomenon to which our agent is confronted in a random situation, therefore providing insights which improve the quality of decision-making? (2) Operationally, doesn't such a confusion between these situations generate a biased representation, unfounded expectations and a lack of accountability? Isn't it thus a source of misinterpretation and governance damaging that negatively affects the quality of decision-making?

1.3 General results

<u>The logical fallacy</u>

Did you ever try to explain intuitively the meaning of "expected value" without relying on the notion of average? Or a value-at-risk is without referring to the quantile that would be observed if the game were to be played several time? Even in a thought experiment, we cannot interpret the concept of, say, expected value otherwise than as embodied in an average[3] – that is referring not to a single event, but to a whole set of events.

Consequently, in a random situation, all reasoning based on an expected value or any other statistical quantity is based on an implicit condition that is impossible to fulfill.

➔ Result 1: *in a random situation, all reasoning based on an expected value or any other statistical quantity is flawed.*[4]

Biased apprehension of the situation

When making a decision with the help of a type 1 function, the intuition we call on to understand the tool we are using rests on the assimilation of type 1 into type 3. As a consequence, in a random situation, the decision-maker using a type 1 description of the situation is led by this "frequency" metaphor to use a system of thought where the phenomenon repeats and thus to enter into a mental representation where the situation is deterministic. It gives him the feeling that he *knows* what is going to happen: the use of statistical tools corrupts the observer's apprehension of the situation, inducing a perception of the risk models as predictive – and generating afterwards frequent disappointments against these risk models "that hadn't foreseen that…".

For example, the expected value is thus wrongly perceived as an estimator of the result. Such fallacious perception can be illustrated in the following exchange,

[3] just as, mathematically, expected value can be described only by a sum, be it continuous or discontinuous

observed in fieldwork during a large financial company's investment committee meeting:

- CFO: *"we have a higher level of risk than the market [than our competitors]: our equity allocation is higher, our real estate allocation is higher…"*
- A critical participant: "*in that case, we should have a higher rate of return. I don't think that we're really above it*".

The expected returns – higher than the market standards in the risk/return framework chosen by the company – is seen here as an estimator of what *should* have been obtained. The participant thus considers it abnormal (i.e., unnatural, a reason for suspicion, a hint of an error or a lie) that the observed returns are lower.

The obliviousness of users as to the change in the nature of their models explains how certain CxOs (CFO, CRO, or Chief Investment Officers) who steer major financial actors are convinced that their Solvency II risk models are "predictive" (and maintain their position when opposed), even though this is contradictory with the very concept of "risk model". As a direct consequence, they base their decisions on erroneous mental representations, where the risk has disappeared *de facto* from the decision-maker's field of perception. More broadly, this kind of bias in the representations we make of the world leads to making decisions related to randomness on the basis of a cost-effectiveness analysis, such as when signing a reinsurance treaty, a CFO or CRO compares its price to a deterministic "cost of risk", or when an investor considers the Sharpe ratio or one of its embodiments. This kind of practice marks the final stage in the process of drifting from the apprehension of risks to a quantified risk/return couple, where risk is formally absorbed into a single synthetic performance indicator. Risk has thus completely disappeared from the decision-maker's perception of his environment, leaving him with a control panel that has been reduced to one figure and stripped of all randomness.

➔ Result 2: *By assimilating random situations into heterogeneous situations, the use of statistical indicators leads to a deterministic perception of phenomena and therefore to a distortion of our apprehension of the world, out of which the risk paradoxically tends to disappear.*

<u>Act of faith and absence of responsibility</u>

In a heterogeneous situation (which is to say, when we are faced with function 3), the situation is deterministic and will transform into function 4. It will be possible to measure the gap between the two and assess the modelling error. Whenever it is the case, the *ex ante* description can be proved wrong, which is the characteristic of a scientific approach (Popper, 1959). On the contrary, for an agent in a random situation, the distribution function cannot be back-tested since the agent will always be faced, *ex post*, with a Dirac-type distribution function. Furthermore, having the function 1 *ex ante* authorizes any function 2 *ex post*: Therefore there is neither way to define and measure the potential modeling error, nor to prove the model wrong. In such a situation, relying on a "risk model" is an act of faith – not science, in a world of irresponsibility – not accountability.

➔ Result 3: *While modelling a heterogeneous situation is scientific and committing, modelling a random situation through a distribution function (or related statistics) is an act of faith with no accountability, as its quality cannot be assessed* ex post.[5]

[5] As regards risk models, the notion of *model error* – most often referred to as *model risk* – has been the subject of great interest since the crisis of 2008. Yet the result above can be reformulated as follows: when one observes only one outcome, he cannot distinguish which part of it would be the random deviation that was modelled and which part of it comes from a modelling error. They are fully fungible; as such, risk models – being indistinguishable from their associated model risks – do not exist in a random situation: *while the concept of the* dispersion model *makes sense in a* heterogeneous situation*, the very concept of a* risk model *is not operationally pertinent in a* random situation.

1.4 Summary of the theoretical framework

We have laid out the theoretical framework that allows us to distinguish two kinds of different phenomenon: random situations and heterogeneous situations.

(1) When a decision-maker is confronted with a random event that will occur only *once in the context of his decision-making*, he is in a *random situation*. If, *within the context of his decision-making*, this event occurs an *infinite* number of times, he is in a *heterogeneous situation*.

(2) In heterogeneous situations, the decision-maker has an almost deterministic understanding of the world: he knows *ex ante* the distribution function of the events he will observe *ex post*. This is not the case in random situations.

(3) This alternatively predictable or unpredictable character of the result is what allows us, more generally, to demarcate heterogeneous phenomena from random phenomena in non-polar cases where the number of future realizations is more than 1 but is not infinite: *if the forecast error* for the distribution function observed *ex post* by the decision-maker *is negligible in his eyes, it is a heterogeneous situation. If not, it is a random situation.*

In random situations, the reliance on statistical quantities is based on a conflation of randomness and heterogeneity. The principle consequences of this are as follows:

In a random situation,

(1) To base a decision on the statistical quantities associated with the distribution function expected *ex ante* relies on a fallacious reasoning. Such an analysis alters the decision-maker's perception of the phenomenon, giving him an illusion of determinism, skewing his reasoning and impairing his ability to take an appropriate decision;

(2) Risk model and model risk become indistinguishable; it is impossible to determine accountability regarding the model's quality *ex post*.

Legal heuristics tend to demonstrate that our societies are used to differentiate between situation of risks and situation of heterogeneity (see appendix A). However, some regulations in insurance tend to conflate the two. The two following parts aim at clarifying such misunderstandings and build up better bases.

2. Fair pricing

2.1 The regulatory debate

Rates have historically corresponded to the "price of risk" of rate classes, to which a margin is added. With progress in genomics, the rise of social networks, the arrival of connected objects, the development of information technology, and advances in the data sciences, big data enables increasingly finer segmentation: from a technical perspective, it refines the pure premium according to individual risk, and from a commercial viewpoint, it refines the margin according to the individual willingness to pay. Faced with this change in the landscape, European and American societies, driven by the same desire to avoid differentiation considered undesirable (undue discrimination), have reacted in opposite ways.

Europe limits possible differentiations of the pure premium. For example, albeit women have a higher life expectancy and drive more safely, European regulation prohibits ratings based on sex in motor insurance or even pension: "*In order to ensure equal treatment between men and women, the use of sex as an actuarial factor should not result in differences in individuals' premiums and benefits.*" (European Council, 2004; confirmed by the Court of Justice of the European Union, 2011). In this view, fairness includes equity and leads to limiting the trend of pricing at "the price of one's risk".

Conversely, the Consumer Federation of America (CFA, 2013) wants to stick to the pure premium approach: Indeed, he seeks to prohibit price optimization – that is the fixing of margins based on the willingness to pay – stressing that "*moving pricing away from its historic cost-based approach*" is "*unfairly discriminatory*". He receives strong support from some authorities, e.g. from the Insurance Commissioner of California, who states that "*a fundamental tenet of actuarial standards, [is] that rates be based on the expected value of all future costs*" and that "*In California […] there are no differentials allowed based on whether the applicant or insured is more or less likely to look elsewhere for a lower price; we would consider such distinctions to be unfairly discriminatory*" (Jones, 2013). This position follows a long tradition that judges the pure premium to be the frame of reference for determining the "fair price" (Daston, 1989; Feller, 1968, p. 249) and is nowadays widespread.

Given this opposition between, say, the EU and the USA, one might wonder whether pricing based on the pure premium represents indeed the "fair price"? We will argue that such perception derives from on a conflation: while it indicates indeed the true cost for a group of insured *from the perspective of the insurer*, it is in no way the "fair price" schedule for the policyholder. Such a "fair price" can only result from the elaboration of principles of justice, which by far exceed the technical calculation of risk premiums.

2.2 The critical analysis

On expected value as the fairness reference framework

While the insurer is in a heterogeneous situation, the policyholder is in a random situation. Expected value is a pertinent tool *for the insurer*, for whom it is embodied in an average on his overall portfolio (if the models are good); but there is no reason *a priori* to transpose this reasoning onto the policyholder's situation by considering that the cost for the producer would correspond to the "fair price" for the consumer. Indeed, the expected value is nothing but the average cost for the insurer. When compared to pricing practices in other industries, in many instances the price that a customer pays (and is willing or ready to pay) is not the production cost; for luxury goods, it is much higher; for subsidized products (e.g. railway or farming products), it may be much lower. Our societies do not consider prohibiting such practices as "unfairly discriminatory". Then, why would it be so in insurance? Would there exist a peculiarity of insurance industry aligning the customer value on the producer's cost?

Beyond the conflation between the insurer and the policyholder viewpoints, the perception of pure premium as the fair price of risk is grounded in the economic vision of expected utility (von Neumann and Morgenstern, 1944) as the descriptive and normative standard of a rational agent behavior. Following this theory, the appropriate frame of reference to assess an agent choice under uncertainty is the statistic expectation. However, albeit widely shared, this approach has been dismissed. Regarding the consideration of expected value, Allais (1953) for example underlines that it "is obvious that, *if it is a question of a single dice throw that will not recur, the justification founded on the law of large numbers does not apply at all.* However, and precisely, *most cases where we have to make random decisions are isolated cases. There, in our sense, the law of large numbers has not any utility. […] There is in no way a code of conduct which should be considered more rational than another one*".

From a descriptive viewpoint, the expected utility representation raised growing empirical concerns leading to distort the standard model to grasp what were considered as cognitive or psychological biases[6]. Decision theorists have since come to the conclusion that no model built on these bases can successfully

[6] e.g. Friedman and Savage, 1948, or Kahneman and Tversky, 1979. Or, more generally, all attempts to distort either the utility function or the probability function with subjective probabilities; or, lastly, as numerous models that were built which, albeit belonging to the theoretical field called "*non-expected utility*", rely heavily on expected utility, e.g. *rank dependent expected utility*, *maximin expected utility*, or *Choquet expected utility* .

combine the necessary answers to the different observations in one global theory[7]. Hence considering expected value as referential to apprehend a random situation, is neither a rationality criterion[8] nor an apt description of human choice.

Since the expected value is meaningless in a random situation, the pure premium has no specific legitimacy *from the viewpoint of the policyholder*. Hence, from the viewpoint of a male driver, it is possible to claim that gender segmentation, although accurately reflecting the costs of the insurer, is discriminatory. From *his viewpoint*, such postulated "fair price" does not correspond to a criterion of justice.

On expected value as an objective measure

Furthermore, considering the individual price of risk as the fair price from the policyholder viewpoint relies on an implicit operational requisite: the *exact individual* probability of claim exists and can be asymptotically reached. Let's consider an example: the probability, for a given person, to suffer from a cardiovascular event, and the progressive attempts to refine it. The first step is to consider the probability over the global population. The second step leads to take into account that it is a female: the probability becomes lower. The third step consists in taking into account the progress of medicine and identify that she drinks a lot: the probability becomes higher. The fourth step relies on the invention of connected devices measuring physic activities: she does sports and the probability decreases. The fifth step builds on new progress of medicine and technologies: it is now known that stress increases the risk, and stress exposure

[7] e.g. Camerer, 1989, or Barberis, 2013. For concrete and recent examples of observations disagreeing with standard expectation based theories, see for example Barreda-Tarrazona et al., 2015.

[8] Regarding the axiomatisation of EU theory by Savage (1954), Gilboa (2009) demonstrated that Savage's axioms do not correspond as such to rational decision criteria.

can be assessed through smartphone data (busy diary, frequent mails identified as aggressive through text mining, etc.), the probability increases again, etc.

Two points are here crucial: (1) there is no reason for these successive evolutions to reach an end since however refined the segmentation, there is always subsidies within a given segment, and (2) there is no reason for the amplitude of these evolutions to decrease. Consequently, *from the policyholder viewpoint*, the N^{th} estimation of the probability cannot be considered as "more accurate" than the $N\text{-}1^{th}$ estimation, as the $N+1^{th}$ is susceptible to contradicting it, both in direction and in amplitude[9]. In other words, *in the policyholder viewpoint*, increasing segmentation is not a progress drawing him closer to his "real" cost of risk.

2.3 The alternative operational solutions

A policyholder's demand to "pay the price of his risk" thus appears both unfounded and impossible to realize in practice[10]: imposing limits on technical rate differentiation cannot be contested *in the name of an individual ethics*, and promoting, as a matter of principle, a rate based on the cost of risk as being "fair" appears moot.

Our observations could be mustered to defend a libertarian vision of pricing regulation – that is, no regulation. As a matter of fact, following our

[9] This observation relates with the reference class problem, which was already discussed at length in the 1930s (McGoun, 1995), and has since been recalled, from Reichenbach (1940) to Eagle (2004). This is a direct consequence of the random/heterogeneity conflation as, in a random situation, the reference class issue becomes an impasse. As von Mises (1949) pointed out, a unique event is never a member of a class. And, even if an objective calibration was possible, he underlines that "*It is implied that this ratio 9 : 1 [the probability] tells us something substantial about the outcome of the unique case in which we are interested. There is no need to repeat that this is a mistaken idea*." In practice, when the number of future realizations of a phenomenon is low for a given observer, the total number of observations (both past and future) for this observer is generally low. This is either due to the total number of realizations, regardless of the observer, being low or because, on the contrary, the phenomenon in question can occur in a heterogeneous situation, *but for other observers*. As a consequence, the reference class issue is more intense: as von Mises (op.) pointed out, a unique event is never a member of a class, and therefore, in a practical random situation, quantification does not allow to realize the practitioners' usual ambition of objectivity.

[10] And actually, when explicitly emphasized by a policyholder, this argument is generally nothing but a way to express the desire of paying not a fair price, but a lower price.

observations, there is no more argument of fairness-towards-individual-customers to be opposed to the freedom of pricing for companies, and to the reduction of moral hazard through a tariff adapted to the consequences of anybody's behavior.

However, from a practical viewpoint, such position would lead to a clearly sub-optimal collective situation. For example, regarding bodily injury insurance, Israel switched from a state-owned no-differentiation in the 80's[11] rate schedule to a private companies system authorizing segmentation on a given set of criteria, within certain limits. It soon became clear that some segments, such as young drivers, were largely underpriced even at their higher limit. Yet bodily injury insurance is both a compulsory insurance and a social welfare necessity. In terms of fairness, pricing according to the risk premium would mean an unaffordable insurance for younger drivers. Such a situation shows the limits of the pure premium principle in terms of fairness, even at the collective level.

Hopefully, as our analysis cuts the relationship between pure premium and fair price, it also represents a strong support for the partisans of a more balanced position, refocusing the debate over pricing discrimination on the question of its economic efficiency, primarily the balance between exclusion and moral hazard (see Dionne and Rothschild, 2014, for a more thorough analysis). In such vision, the fair price is nothing other than the competitive price on the *regulated* market – i.e. *market with potential technical or commercial differentiation prohibition for social purposes.*[12]

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[11] No differentiation except for the engine size.

[12] Regarding the scope of regulation, when making decision between economic efficiency and social justice in order to define a set of prohibited differentiation criteria, there is no reason to distinguish between commercial differentiation ("price optimization") and technical differentiation. Indeed, as pure premium is not a relevant framework of reference anymore, it does not matter whether these criteria have an impact on the cost of risk or on the consumer's willingness to pay.
Furthermore, with no pricing regulation, the price would depend on the fact that, focusing on anything but insurance issues, somebody developed technology A rather than technology B.

3. Capital requirements

3.1 The regulatory debate

The last evolutions of the main prudential standards revealed distinct choices. The RBC system built in the 90's in the United States, as well as Solvency I (which was consolidated in Europe until the early 2000's), albeit taking into account a hierarchy of risks[13], does not pretend to determine the capital requirements as a risk measure of the own funds.

By contrast, for the past fifteen years, the European Union has been implementing a new prudential regulation in the insurance sector (European Commission, 2009) that, in its quantitative dimension, pretends its capital requirements to correspond to *the* annual VaR of 1/200 of the own funds. This regulation is often perceived, or presented by its promoters, as cutting-edge. However, relying thus on a statistical measure to assess the solvency of a company – that is for strategic risk management purposes dealing with the forthcoming year survival or bankruptcy of a company, is basing the regulation on a conflation between heterogeneity (statistics on the whole industry or over a long period) and randomness (dead or alive at the end of the year). As a consequence, the set of general results which we established in Part I applies and we will successively present the operational embodiment of results (i), (ii) and (iii).

3.2 The critical analysis

<u>The interpretation and calibration issue</u>

Thus, rate differentiation would depend on criteria haphazardly emerging rather than on collectively debated and chosen criteria.

[13] Solvency I does so in a very rustic way, only distinguishing UL from minimum guaranteed rate contracts in Life and liability insurance from other lines of business in non-life. The American RBC standards are based on finer characterization of the risk, including per asset types.

The regulation fixes the calibration of the capital requirements to the 99.5% VaR, which seems to be mathematically univocal. But how should we interpret that? As always, interpreting a probability measure requires to posit oneself in a heterogeneous situation: some say that with such a regulation, no more than one company out of 200 will collapse each year; others say that a given company will be able to face a stress which period of return would be 200 years.

These two interpretations are not equivalent as they generate different calibrations. If we expect approximately one of every 200 companies to go bankrupt each year, then we should look at the dispersion of the different companies that make up the industry and review the calibrations annually. If we expect that each company statistically goes bankrupt one year out of every 200 years or so, then we should rather look at the dispersion of the situation in time and the calibration, company by company, should remain stable over time. In terms of objective, the first option would relate to idiosyncratic risk and correspond to "*the traditional solvency risk controlled by the existing solvency regulation that are aimed at counterweighting the moral hazard problem*" (Bobtcheff et al., 2016) whereas the second option rather corresponds to a kind of systemic risk management, and interacts with the procyclicality issue (Frezal et al., 2016). In terms of concrete calibration, none option makes sense: the first option would require to measure the worst deviation between 200 firms, and considering it for dissimilar companies would be meaningless; the second option would lead to measure the worst deviation over two centuries, a period covering so disparate years.

Mathematics of randomness and of heterogeneity are the same, so the regulatory quantum seemed univocally defined, but as randomness and heterogeneity are distinct issues the creators of the regulation did not address, the targeted scope of heterogeneity has never been defined: nobody knows whether the operational interpretation and calibration of the quantile should be temporal (option 2) or “geographical” (option 1). The objective is unclear, the interpretation equivocal and the calibration undefined. As a result, the European regulator used an inconsistent mix of the two options: it calibrated the non-life underwriting components of the standard formula following the first option (CEIOPS, 2010)[14] and the market risks following the second option[15]; regulation is not technically sound. Where the political level had legitimacy to make choices between, say, economy funding, product development and policyholder protection, it delegated it to the technical level, for nothing.

A regulation distorting apprehension

From a general viewpoint, the fact that banks and their regulators tended to rely excessively on quantitative models before the 2008 crisis (Ashby, 2011) can be explained by the fact that they were accustomed to using successful models in a business/pricing context. When these models were transposed onto a steering/risks context, the nature of the models changed – they transformed from predictive models (heterogeneity) to non-predictive models (randomness) – and this change in their nature has not been perceived by their users nor by their environment.

Prudential measures set up by Solvency II, as risk measures, are, by definition, not predictions and thus may generate such kind of serious issues. The observed evolution of own funds at the end of the year will be higher (often) or lower (hopefully seldom) than the capital requirement. However, as already mentioned, fieldwork led to observe several CxOs (both CFOs and CROs) explaining that their prudential models were predictive. Believing that the figures you use are

[14] The methodologies used to determine the stresses for premium and reserving risks, which depend on the lines of business, may include methods which take into account the dispersion of combined ratios or reserving result between the different companies on the European market.
[15] The calibrations are based on the financial markets variability in time and are not revised annually, apart from the equity stress parameter which was decreased in QIS 5 from 39% to 30% to mitigate procyclicality.

accurate predictions of the future means that you consider the world you live in as deterministic, and thus that you are not well equipped cognitively to judge the situation.

Indeed, the use of such tools by experienced professional thus positions them in a fallacious state of mind; the risk tends to disappear from their unconscious steering dashboard: decision making under uncertainty appears no more as a matter of strategic choice but only as the mechanistic optimization of a ratio (New Business Value divided by Solvency Capital Requirement (SCR) to decide whether or not to launch a product, expected result divided by gain in SCR to decide whether or not to enter a reinsurance program, expected return divided by increase in SCR to decide whether or not to invest in riskier assets, etc.). When calculating risk in that manner, the consciousness of consequences, which is so decisive in the risk return trade-off, disappears, just to be replaced by a new definition of the return – a "risk adjusted [expected] return" which will concretely never be observed.

Towards the end of accountability

In her paper on risk management in banking industry, Mikes (2011) proposes a classification of CROs in two categories: "quantitative enthusiasts" and "quantitative sceptics". She explains that while the quantitative sceptics tend to seek impact on management choices and feel committed if things go wrong, the quantitative enthusiasts rather consider that they did their job providing a risk quantification and that they are in no way liable for the decisions taken.

This can be explained by our framework, as prudential regulation asks CROs to deal with major potential adverse events endangering the survival of the company: regarding these events, top management is in a random situation. First, Mikes' observation is thus a direct and natural consequence of our general observation of the fact that, in a random situation, no risk model can be proved wrong: when positioning himself as the top-expert of risk measurement, the CRO sets up himself in a situation where it becomes impossible, *ex post*, to reproach him with having made a technical mistake[16]. Second, it may be the consequence of the fact that, in a random situation, no adverse outcome, even extreme, is formally excluded by any risk measure – it is only estimated as highly improbable: even if positioning himself as a decision-making participant, the quantitative enthusiast CRO acts *de facto* as a formalizer of "bad luck", offering *ex post* a justification of the decision as "optimal given what was scientifically known when the decision was taken, even if it proved to be catastrophic afterwards".

Furthermore, our observations during a fieldwork in a major insurance company extends this propensity of risk measures to make people no longer accountable for their actions to the managers in the business units. For example, implementation of Solvency II acted as an argument for operational decision makers to no longer care about risks: when the risk department representatives challenged them on the risk dimension of a new product launch or an investment decision, their reaction regularly was: "*you [risk department] calculated risks and we [the company] have capital to face it, so why, on top of that, do you ask me to care about it? Do you mean that your risk modelling is inappropriate?*"

3.3 The operational solution

The problem to be solved

The main problems arising from the use of statistical quantities to apprehend randomness when regulating a company, are the following:

16 This can be observed, in particular, as regulation authorizes companies to use internal models to determine the 1/200 VaR while the company is in a random situation in regard to its ability to survive. Due to the porosity between the risk model and the model risk and the resulting lack of accountability in such a situation, one cannot scientifically objectivize *ex ante* nor *ex post* the adequacy of those internal models.

(i) A regulation as Solvency II is grounded on quantitative foundations that are both equivocal and technically complex, where the underlying choices do not correspond to a concrete interpretation which the political level could grasp. In other words, while the calibration of the required resistance is supposed to be the result of an assumed political trade-off with issues other than prudential, the measure, being not interpretable, has been *de facto* delegated to the technical analysts, so that the decision has not been made at the right level.

(ii) As soon as the risk is purportedly quantified, and subsumed in one figure, it is no longer perceived as a risk. Thus, the Solvency II indicators, which are used to steer the companies, skew the decisions by distorting the decision-maker's perception of the situation.

(iii) Moreover, given that a risk-taking decision-maker is considered rational if and only if he uses the results from such types of calibrations, and that the quality of these comprehensive statistical calibrations provided by the analysts can never be evaluated *ex post*, it is no longer possible to establish accountability for the quality of the decision-making.

In order to avoid these flaws, the tools on which to regulate and base a decision under uncertainty must allow for *the feeling of randomness to be reintroduced, so that the perception of the random nature of the situation remains* and for *the decision not to be driven by uninterpretable analysts' subjective choices, so that it is taken at the right level*.

<u>Towards a solution</u>

A structured scenario-based approach allows answering these issues. As capital requirements are a key indicator when steering a company, the design of capital requirements should be aligned with the strategic risk management processes considered appropriate. This advocates for a two-layers system:

- The first layer would provide an answer to point (i). It would consist in defining a set of scenarios (financial, underwriting and others, eventually intertwined) at the regulatory level, devoid of reference to any VaR indicator. Capital requirements would correspond to the amount of capital needed to survive any of these scenarios. The choice of this set of scenarios would allow to consider the political trade-off between prudential issues and other economic, macro-financial and social issues.
- The second layer would provide an answer to points (ii) and (iii). It would consist in defining a set of *ad hoc* scenarios at the company level, by the top management, and check that they would not generate bankruptcy. These scenarios would be defined as the scenarios (1) which occurrence the top management would consider not negligible and (2) under which the company should, *given its risk appetite*, resist[17].

In this framework, the strategic risk management decision, both at the regulatory level and at the company level, resides in the definition of the set of scenarios. This set respectively defines the risk appetite of the society (regulatory level) and of the company (top management level): should the company be able resist to a home country govies default, or do we accept bankruptcy in the case of such event? Should it resist to a Great-Depression-like financial crisis? To a Spanish-flue-like epidemy? To a combination of these two last events?

Cons and Pros

Renouncing statistics requires that we renounce (i) the ambition to compact the potential outcomes to the point of reducing them to one or several deterministic statistical indicators that claim to synthesize all possibilities and (ii) the ambition to manage risks through a purely technical analysis, with a dichotomy between risk calibration experts and decision-makers. Furthermore, from a psychological

[17] The second layer is a kind of *Major Risks Self Assessment* exercise, as exists in the Pillar 2 (qualitative requirements) of Solvency II.

viewpoint, this process is harder to bear for the decision-maker insofar as he must take responsibility for the arbitrary aspect of his decision – defining, as a negative, the scenarios under which he accepts to get bankrupt, and cannot shift blame onto a hidden technician or an ethereal statistic.

However, such a process is intuitive as it relies on an instinctive decision-making process. As a matter of fact, according to Solé (2000), individuals make their decisions by obliviously categorizing future scenarios into three classes: that which cannot occur (Impossibles), that which could occur (Possibles), and that which is inevitable (Non-Impossibles, which we cannot imagine not occurring). These mental representations are largely the unconscious result of each individual's culture and experience. They define our perception of reality: a mental schema of analysis that frames decision-making for each of us[18]. Our proposal is nothing but a formalized – explicit and thus generating awareness and accountability –, process of this oblivious decision-making schema: the set of scenarios which are to be determined by the regulation and the top management reflect their strategic view on the worst "Possibles" to which the company should resist.

Naturally, our methodology is therefore in line with some heuristics occasionally privileged by professionals who are intuitively used to consider that a statistical profitability analysis would not be pertinent when facing randomness. As such, one technique used in choosing a reinsurance treaty, for example, is building nomograms {number of serious claims, cost of these claims} and identifying the

[18] Solé takes the example of the Pearl Harbour attack: at a dawn of December 1941, the radar operators in Pearl Harbor observing points on their screens saw a dysfunction of this recent technology as a Possible, and an attack without declaration of war as an Impossible; such a representation of the future led them to spoil two hours checking their radar rather than alerting and drifting the ships out of the harbor. This representation of reality defines the strategy and is therefore necessarily subjective (obviously, the Japanese pilots, at the same moment, did not have the same representation of the future) and, as a consequence, is the prerogative of these who are legitimate to arbitrate trade-offs: top-managers and politicians.
In the Appendix B, we propose a mathematically formalized description of this decision making process.

main areas that appear in order to determine how to cover for risk in accordance with how this corresponds to "scenarios" that seem qualitatively credible or not.

Our approach also corresponds to a focus on the possible consequences of our decisions, which also corresponds to Taleb's recommendations (2007)[19]. As such, in terms of asset allocation, a 1/N allocation strategy such as Taleb (op. cit.) or Haldane and Madouros (2012) recommend corresponds to not wanting to exclude from the field of Possibles any scenario in which a given asset's value falls. The old rules of diversifying investments to which European insurance companies cleaved in Solvency I, and which limited the proportion of the balance sheet invested by issuer category and by issuer, are an embodiment of this. It also corresponds to the popular wisdom that has filtered into the proverb "don't put all your eggs in one basket" in several languages, and not "put all your eggs in a quilted basket" (toward which risk-based but ban-free regulations, such as Solvency II, tend).

4. Conclusion

"[To make] a strenuous effort to put aside some of the artificial schema we interpose unknowingly between reality and us. What is required is that we should break with certain habits of thinking and perceiving that have become natural to us. We must return to the direct perception"

H. Bergson, *La Pensée et le mouvant*, 1938

(Second lecture at the University of Oxford, 1911)

Tools that damage the quality of decision-making

The economic and financial tools for apprehending risks on the basis of statistical indicators are founded on the conflation of two phenomena of

[19] but for different reasons. Taleb considers that we cannot know the probabilities of rare events; we judge that as soon as we decide to consider their occurrence as possible, their probability is not an issue.

different natures: heterogeneity, for which statistical concepts have emerged and in reference to which we continue to intuit these concepts, and randomness, onto which these concepts have been wrongly transposed.

The pertinence of certain tools is contingent on the observer's position in respect to the phenomenon he seeks to understand. The transposition onto random situations of tools conceived for understanding heterogeneity is exceedingly easy from the standpoint of mathematical formalism, but it is based on the identification of two decision-makers whose positions are radically different. For observers in a random situation, this transposition, which leads to the use of statistical indicators,

(i) is not pertinent because it relies on a logical mis-reasoning,
(ii) gives rise to an erroneous mental representation of the situation by creating an illusion of predictability,
(iii) and does not provide the means to make the actors aware of their responsibility.

Hence, the "information" provided is likely to harm the quality of the decision-making.

<u>Regulatory consequences</u>

Common visions over fair-pricing and prudential regulations (together with quantitative risk management practices) are based on the focus on or optimization of statistical indicators (expected value, variance, VaR, etc.) and are therefore subject to these failings.

Regarding fair-pricing, our analysis dismisses expected cost and its diverse estimations as a relevant frame of reference for a client choice. As a consequence, it advocates for a pricing regulation:

- which does not prohibit *per se* a price optimization on the argument that it would take the rate away from the pure premium,
- and which feels free to prohibit the use of some differentiation criteria to reach a social equilibrium balancing efficiency with solidarity issues.

Regarding prudential regulation, our analysis dismisses the Solvency-II-Pillar-1-like capital requirements as a relevant regulatory framework. Indeed, the reference to the 99.5% VaR indicator blurs the real issues behind a technical, and – as we have tried to demonstrate- fallacious approach. It displaces the debate to a chimeric statistical field which is in fact neither more objective nor more legitimate. The set of common scenarios defined by the public authority should invoke instead an accepted responsibility on risk-taking, and the legitimate arbitrage between the protection of the policyholders and the multidimensionnal impacts of any financial regulation in the economy. This approach thus advocates for other procedures to be employed by regulators such as stress test approaches. It is in accordance with the emphasized importance of ORSA to assess the risk profile of a company (Fischer and Schlütter, 2015), this time at the macroeconomic level. The crucial point here is that the *actual* decision is made when defining the set of scenarios the companies should resist to. As a consequence, this methodology not only improves the pertinence of the decision-making framework, but also prohibits actors from abdicating responsibility.

<u>Fields of impact to explore further</u>

From a theoretical standpoint, the distinction between randomness and heterogeneity provides a rationale to explain some legal heuristics, such as the apparent paradox of the coexistence of prudential regulations, which aim to prevent financial institutions from taking as many risks as they like, and of limited liability companies, which on the contrary encourage them and their controlling shareholders to take more risks by protecting them from being held responsible for unlimited losses (see appendix A).

From an accounting standpoint, this distinction leads to question the relevance of stochastic Present Value of Future Profits (PVFP), stochastic Best Estimate of Liabilities (BEL) and Time Value of Option and Guarantees (TVOG) when valuating a life insurance company balance sheet. Indeed, while an insurer faces the heterogenity of numerous contracts on the underwriting side, he is in random situation regarding the financial markets evolution. As a consequence, while considering the expectancy to set-up the reserve is relevant as regard underwriting parameters, considering expectancies among different financial scenarios rather than a central scenario, as it is done when calculating a TVOG, could appear meaningless.

From an operational standpoint, the fruitfulness of this distinction in the field of risk management must be explored further. Regarding financial regulation, for example, it can suggest evolutions in governance. Indeed, activities concerning the calculation of capital requirements are generally attributed to the CRO, due to their claim of capturing risks. However, this calibration rests on a foundation shared with profitability measurement tools and is arbitrary from the viewpoint of decisive risks. As such, this would seem to argue in favor of such responsibilities being steered by the CFO along with the other balance sheet parameters. Conversely, as risk management necessitates the imagination of possible futures, it would make sense to entrust strategy function to CROs.

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Appendix A: Legal heuristics which tackle this conflation

As a risk that is too high for an individual can thus be constitutive of a heterogeneity that is beneficial to society and vice versa, this distinction helps to explain certain juridical mechanisms as heuristics developed by our societies to distort individual decision-making through mechanisms that alternatively curb or promote risk taking. In the first category, we find examples such as prudential regulations, which aim to prevent financial institutions and their executives from taking as many risks as they like so as to limit bankruptcy. In contrast, the second category includes examples such as limited liability companies, which protect executives and shareholders from being held responsible for unlimited losses and protect them from the claims of creditors in the case of bankruptcy. What is the difference between these two categories? In the first case, it is the savers who suffer the consequences of bankruptcy, and in the second, it is the suppliers. Savers generally do not open several retirement or savings accounts – and anyway the financial system is exposed to systemic risk: they are in a random situation regarding the possible bankruptcy of their bankers or life-insurers. The issue is thus no longer expected value: the possibility of ruin would be too painful for the savers, so it is necessary to limit the potential and the impact of such an eventuality. By limiting risk taking that would be optimal for the shareholders of the regulated company (and who diversify their investments, thereby placing themselves in a heterogeneous situation that may push for each company to take risks), society protects the savers, who are in a random situation. Conversely, if a standard non-financial company goes bankrupt, it is said company's suppliers who will not be reimbursed and will lose a part of their revenue. Most suppliers, however, are in a heterogeneous situation: statistically, they already know that they will suffer losses due to one client or another, and a gains/losses balance can then be established socially. Such a balance is effective and must not be disturbed by an executive, who would be in a random situation if he had to make up for the bankruptcy with his own assets. Society limits this potential disturbance of social efficiency by limiting the small business executive's responsibility. Such comparisons raise the question of whether or not it is appropriate to maintain a limited liability for shareholders and top managers of financial institutions.

Appendix B: Mathematical formalization of an *à la Solé* decision making process under uncertainty

Let us propose an intuition of the process of analysis and decision, which is based on the following steps:

(i) Identification of all possible states of nature: Opening the decision-maker's field of potential Possibles.

In the case of an asset allocation decision, for example, a typology of possible scenarios could be: (1) rise of the spread of government bonds accompanied by a drop in stock prices and a rise in property prices; (2) negative rates accompanied by a rise in stock and property prices; (3) etc.

(ii) After the most exhaustive inventory possible: The rejection of those scenarios that would be considered “Impossible” *by the decision-maker* so as to retain only *his* “Possibles”.

By “Impossibles”, we mean those scenarios for which the decision-maker does not want to take into account the possible realization either because he judges it to be negligible (literally Solé’s Impossibles) or because he accepts to assume the risk (we here widen his acceptance). In the case of an asset allocation, for example, the decision-maker would have to take a position on whether or not he considers a default of his country’s govies as an “Impossible”.

(iii) The analysis of the impact of various possible actions in each of the remaining “Possible” scenarios: The choice of the action that maximizes the decision-maker’s utility in the scenario in which he “believes” most, under the condition that the consequences are acceptable in each of the Possible cases. In our investment example, this would consist in choosing the allocation that maximizes the returns in the preferred scenario, under the condition that these returns remain acceptable in his other Possible scenarios.

Such a process may surprise because it does not make use of probabilities as such, only the parameter “negligible or not”; but from a normative perspective, it is consistent with the lack of meaning of statistical indicators (such as probabilities) in a random situation, and from a descriptive viewpoint, it seems a natural fit with Solé’s analyses[20].

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[20] Of course, this contradicts Arrow’s apologue (1951), which uses the following example to justify the claim that a decision-making reasoning under risk cannot do without probabilities: “*If an individual were told to predict whether or not two heads would come up in successive throws of a fair coin and further informed that he would lose his life if he guessed wrong, I find it very hard to believe that he would disregard the evidence of the calculus of probability[…] [A]n extension of this suggests that in almost any reasonable view of probability theory the probability of a single event must still be the basis of action where there are genuine probabilities.*” But, beyond the fact that he ignores the central problem of reference class for a

Let us propose a formalized version of this process:

Let S *be the set of all existing states of nature,*

of which S_i *is the set of all states of nature that agent i imagines,*

of which $\overline{S_i}$ *is the set of all states of nature that agent i identifies as Possibles.*

For example, the case of a giant meteorite razing Paris ∈ $\{S - S_i\}$; the case of a centenial Paris flood recurring this year ∈ $\overline{S_i}$ for the homeland security (which is prepared for such an eventuality), but ∈ $\{S_i - \overline{S_i}\}$ for the organizers of Roland-Garros, who certainly accept to disregard the possibility of such an occurrence, or, in other words, run the risk of being unprepared for a flood (and for which nobody would blame them).

Likewise, let A *be the set of actions that a human being can imagine,*

of which A_i *is the set of actions that agent i imagines,*

of which $\overline{A_i}$ *is the set of actions that agent i identifies as Possibles.*

For example, the action of killing one's parents to inherit their wealth faster certainly ∈ $\{A - A_i\}$ because the idea would not even occur to us; according to the person, the case of cheating on one's partner belongs either to $\overline{A_i}$, or to $\{A_i - \overline{A_i}\}$ if, as a matter of principle, *i* considers it taboo; if it belongs to $\overline{A_i}$,

choice made in a more realistic context, this example overly simplifies another aspect of "real life". A choice such as the one proposed by Arrow is between -∞ (to die) with a probability p (1/4) or with probability 1-p (3/4). In a real situation, even a highly simplified one, an individual is not faced with the choice between A with probability p and A with probability 1-p. Let us suppose for example that for your retirement, you can put your 1 M$ savings into a lottery that will pay out 2 M$ if you win. This means that you have the choice between ultimately holding onto 1 M$ with the probability p=1 or playing for 2 M$ with the probability p' vs. 0 with probability 1-p'. If we ask the question "does the result depend on whether p' equals 1/4 or 3/4 ?", Arrow's claim becomes much less self-evident. The only issue at stake here, in deciding whether or not to make the bet, becomes: "can I ignore the possibility of losing my savings?" or, in other words, "does losing belong to what I subjectively consider an Impossible?".

then the question of cheating – which is to say the consequences of the risk – will arise.

Now, let $\dot{S}_\iota$ be the state of nature in which agent i believes most

Let us note that U(a; s), the pay-off associated with the action a, combined with the state of nature s, and $\overline{U}_\iota$, the worst result that i finds acceptable.

The decision making process of agent I then becomes :

(1) *Maximising S_i and A_i (getting the greatest set)*
(2) *Defining $\overline{S_\iota}$ and $\overline{A}_\iota$, $\dot{S}_\iota$ and $\overline{U}_\iota$,*
(3) *Retaining the decision $\dot{A}_\iota$ such that its pay-off $\dot{U}_\iota$ satisfies:*

$$\dot{U}_\iota = \begin{cases} \max\limits_{\overline{A_\iota}}(U(a;\dot{S}_\iota)) \\ under\ constraint\ \min\limits_{\overline{S_\iota}}(U(\dot{A}_\iota, s)) \geq \overline{U}_\iota \end{cases}$$

Let us note here that it is possible to draw a certain number of parallels with some usual concepts of decision theory: the fact that A_i and S_i are only subsets of A and S corresponds to a Simon-type bounded rationality; the subjectivity associated with $\overline{A}_\iota$, $\overline{S}_\iota$, and $\dot{S}_\iota$ is linked to the fact that we consider subjective probabilities in expected utility; and finally, $\overline{U}_\iota$ can be understood as an equivalent to a parameter such as risk aversion.

There is, however, a fundamental difference between the two approaches. The *decision* here does not reside in the mechanical operation of constrained optimization (step 3) as it is implicitly in decision theory. *What we highlight is the fact that the analysis is (the most important) part of the decision: the decision resides in the choice of $\overline{A}_\iota$, $\overline{S}_\iota$, $\dot{S}_\iota$ and $\overline{U}_\iota$ – a choice that is committed and conscious, and not a passive characteristic of the individual.*

This choice is irreducibly a-rational (and subjective). Indeed, when one does not know what will happen, there is no good decision *in itself*.