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Working paper

Alea and Heterogeneity the Tyrannous Conflation

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The Tyrannous Conflation

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Abstract:

The uniqueness of the mathematical formalism that enables description of randomness and heterogeneity has led us to mistakenly transpose insights from one domain onto another, distorting our intuitions and damaging our understanding of risk. Condorcet, Allais and von Mises raised warnings often mentioned anecdotally but which since have become concrete problems due to the scientific and technological advances of the past decades and the present. Bringing these warnings together, we formalize the distinction between the two types of phenomenon (random and heterogeneous) in order to draw general conclusions. Secondly, we present examples of this formalized distinction in decision theory, investments, governmental decisions, insurance pricing, and prudential regulation. Finally, we propose a methodology of analysis and of decision-making under uncertainty that does not rely on the conflation of randomness and heterogeneity, and discuss its comparative advantages and disadvantages.

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Introduction

In 2014, a former quantitative analyst who had been working in a large London bank in 2008 explained:

> "After 2008, we had to account for 2 billion in losses. That is to say, we had to find models that would have enabled us to avoid those losses. We tested several complex models – stochastic volatility…and it didn't work. We realized that we'd get creamed every time."

Could the analyst and his colleagues have succeeded in their mission? Would more sophisticated models have enabled the finance industry to avoid the losses incurred during the crisis? In our view, their attempts were doomed to fail.

The objective they were given is symptomatic of an omnipresent confusion in the finance industry, be it in regard to banks or insurance companies, investment strategies or regulation: it is a confusion between the tools used for steering business profitability and those used for managing risk – a confusion between the properties of each and thus between the expectations we can have of each. On the one hand, these institutions, like all companies, individuals, and governments, are subject to uncertainties due to the fact of their very existence and activity. These uncertainties may be adverse, and placing them under control is a question of risk management. On the other hand, the activity of these institutions is to take on the risks of others. It is not a question of managing risks *per se*, but of risk pooling – of managing heterogeneity (of outcomes). As such, a bank will make use of the fact that some of its debtors pay off their debts while others do not, and an insurer will make use of the fact that some of its clients have accidents while others do not. The business model of a financial institution thus rests not on the management of risks but on the management of heterogeneity.

Let's take an example to better understand our financial analyst's (and his backers') error in reasoning by considering two actors: a fisherman standing in the Mont Saint-Michel Bay and the nearby Rance tidal power station. Let us also consider two phenomena: a tide and a tsunami. For the fisherman fishing on foot, the tide is a question of risk management. But for the power station, this is not the case. The turbine is not a tool for managing risk, and, as such, in the case of a tsunami, the tidal power station's ability to survive is independent of the characteristics of the turbine. Would the power station's manager say, "After the tsunami, we tested several turbine models and it didn't work, we realized that we would get creamed every time"? That is exactly the situation our quantitative analyst and his bank were in. Indeed, the pricing models used by corporate and investment banks are profitability models, just like the pricing models in insurance – they are capable of describing heterogeneity and statistically predictive regarding the tide, but they are not predictive models in regard to the randomness of a tsunami. Can they be appropriate tools for managing risk?

Let's return to our example. The tide is a given phenomenon, affecting both the fisherman and the tidal power station, but the fisherman and the power station perceive it differently from one another: while the underlying phenomenon remains the same, they are not in the same *situation* in respect to it. We find a similar scenario in the case of signing an insurance contract, for example. The policyholder and the insurer are faced with the same phenomenon – an unknown regarding whether or not this particular contract will result in compensation – yet the parties are in very different situations: the policyholder, even if he has a certain knowledge of statistics, *does not know* what will happen to him, whereas the insurer *knows* that, statistically, across all its policyholders, it will pay x amount. However, while these two actors are in different situations, we are used to having recourse to the same formalism to describe the common phenomenon with which they are faced when signing the contract regardless of their viewpoint: the distribution function and the statistical indicators describe both the random phenomenon (outcome seen by the policyholder) and the heterogeneous population (outcomes seen by the insurer). Is this valid?

The answer given to us by physics is: potentially no. Indeed, physics tends to adapt its choice of model to the observer's position: ice observed over a short period of time, for example, to build an ice hotel, will require equations from solid mechanics, whereas the same ice observed over a long period of time, such as for analyzing the flow of a glacier tongue, will require equations from liquid mechanics. Conversely, financial institutions tend to use an analytic framework according to the underlying phenomenon *in absolute terms* – not according to the decision-maker's situation. This can lead to transposing tools for situations where they are relevant onto situations where they are not. These tools would thus serve as a vehicle for a cognitive representation of phenomena that is not adapted to a correct understanding of the situation, resulting in a weakened quality of decision-making. This situation affects the fields of prudential regulation, asset allocation, and, more broadly, those of decision theory and public policy analysis.

Furthermore, our glacier example also illustrates that being able to observe and analyze from a global viewpoint does not guarantee a better understanding of a phenomenon. If a pilot wants to land on the glacier, he would be far better served by taking the advice of a mountaineer who has climbed it rather than that of a geologist who, with the distance of having observed the glacier over a long period of time, would advise him to use a seaplane because he sees the glacier as being fluid. Likewise, it is not necessarily a given that the distance (and related knowledge) afforded by statistical technologies and resulting measures is beneficial for a decision-maker in a risky situation. The glacier example suggests that these technologies could conversely weaken his understanding of the situation and the quality of his decision-making.

This paper seeks to position itself at the level of interaction between the phenomenon and the observer/decision-maker, and to identify the situations in which the usual tools and concepts are not relevant for understanding random phenomena, as opposed to those situations in which they are.

Condorcet (Daston, 1988, p. 349), Allais (1953) and von Mises (1949) already occasionally criticized the use of expected value (Condorcet and Allais) and of a probability (von Mises) to describe an isolated phenomenon. We will propose an analytic framework and deal with *any* statistical measure. The first section will define the concepts and accordingly characterize the field of relevance of the above-mentioned statistical indicators (and associated estimators), as well as the consequences of their use outside of this field. The second section will present a typology of fields for which this reasoning is applicable. Finally, the third section will open the way for another tool substitute that provides an alternative to current risk management technologies.[2]

1. An analytical framework demarcating the field of relevance of our tools

"[…] one of the beautiful catches in theoretical physics is to place a symbol or a value that has meaning in Gr into an equation dealing with Gh – or vice versa – and leave it there for

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[2] Let us point out that our goal is not to take part in the philosophical debate concerning the nature of probabilities. We aim to characterize the field of relevance of common decision-making tools – statistical indicators such as expected value, VaR, and risk-return ratio. We will thus limit ourselves to noting that the conceptual framework of this analysis is the concrete framework in which an actor makes a decision: in such situations, the decision-maker seeks to rely on reputedly "objective" elements (in the "factual", "empirical" sense), and the ambition of these tools as well as the intuition their users have of them thus rests on a tacit underlying frequentist vision of probabilities. Beyond this operational aim, such understanding is in line with the vision of probabilities held by most of the major schools of thought in the 20th century. Scheemaekere (2009) presents Kolmogorov as "deeply attached to frequentist interpretation", points out that Popper's theory is a "variant of frequentist theory", and cites von Mises, who states that "the subject of the theory of probability is not speculation, opinions, or other forms of thought, but, in the end, only observable facts as in any other branch of the natural sciences." We consider, further, that a Bayesian interpretation is simply a refinement of the frequentist perspective.

a student to deal with. The chances are enormous that the student falls into the trap and generally remains there, sweating and panting, with nothing seeming to work, till some kindly elder helps him out."
Asimov, *Foundation's Edge*

1.1 Definitions

Our concern here is the relevance of tools (concepts or metrics) that aim to inform decision-making when the future is unknown. We will focus on characterizing the intersection of an underlying random phenomenon and the decision-maker who seeks to understand it. More specifically, we will define this context in a generic way with the word *situation*:

Definition 1: A *situation* is *a context where a future phenomenon and an observer come together,* such that:

(i) the phenomenon will result in one or more realizations in the eyes of the observer,
(ii) before each realization, the observer does not know what will result from the realization,
(iii) the dispersion of the realizations of which the observer can conceive is significant in regard to his decision-making threshold.

For example, taking out an insurance policy relates to a unique underlying phenomenon (whether the policyholder will develop a cancer). The dispersion of the individual outcome is significant, as one's would behave in a different way if he knew the outcome before signing the contract. Notwithstanding the uniqueness of the phenomenon, the policyholder's and the insurer's situations are not identical, as the policyholder signs a single contract and observes a single realization, whereas the insurer signs a large number of contracts.

The insurer's situation differs *operationally* from that of the policyholder in that as the insurer signs a large number of contracts, the underlying phenomenon shall be grasped outside of one isolated case. Taken as a whole, the insurer's portfolio will have a deterministic result thanks to the law of large numbers. With a certain margin of error, of course, the observer can *forecast*. Thus, there is a difference in the *nature* of the observer's situation depending on whether the underlying phenomenon repeats itself or not: in one case, the situation remains random, whereas in the other it is deterministic.

We can therefore identify two ideal types of different natures among all situations:

Definition 2: *random situations* are situations where the observer will only observe a single realization; *heterogeneous situations* are situations where the observer will observe an infinity of realizations.

Let us take two examples in an insurance company: the actuary and the CIO. The actuary, who prices and sells thousands or millions of similar policies, each of which resulting or not in a compensation, does so in a *heterogeneous situation*. Conversely, the Chief Investment Officer chooses the strategic asset allocation for a period that is significant in relation to the duration of his tenure, not knowing what will be the overall return of each asset class: he will make this kind of decision very few times during his tenure, and as such he is in a *random situation*.

We can now rely on the foreseeable characteristic of heterogeneous situations in order to determine the border between random situations and heterogeneous situations when not dealing with an ideal case: we are in a random situation when the number of realizations for the observer in question is "low" and in a heterogeneous situation when the number is "high". The transition area between "low" and "high" can only be defined by the consequences held by its crossing. When the observer judges himself to be in a situation where he can generally make a prediction (a deterministic forecast) and make a decision on this basis while disregarding all other contingencies, he is in a heterogeneous situation. Conversely, when the decision-maker considers the extent of remaining uncertainty to be non-negligible, he is in a random situation. Stated formally,

Definition 2[a]: when the eventuality of a significant deviation from an expectation is negligible in the eyes of the decision-maker, it is *a heterogeneous situation*. In the opposite case, it is a *random situation*.

1.2 Comments

Note that such a definition is based on a perspective that is incompatible with that of Samuelson (1963) in his critique of the "abusive" use of the law of large numbers. He explains that if someone refuses to bet once on a high-stake heads-or-tails game with positive expected value, then, if given the opportunity, he should refuse to bet 1,000 times, as "*no sequence is acceptable if each of its single plays is not acceptable*". In fact, his position – which can be proved, from a logical viewpoint, straightforwardly by recurrence – relies entirely on the fact that a non-zero probability, however small it may be, cannot be considered as zero. We do not share such a position, which would be terrifying from a normative point of view, and which does not correspond to the way we make decisions from a descriptive point of view. For example, we usually make our decisions without taking into consideration the possibility that we might die the following day, as though it were impossible, albeit the eventuality is simply negligible. Likewise, when the light is green at an intersection, we do not stop to check that nobody is coming along the cross street – we know that it isn't mathematically impossible that this would be the case, but we agree to overlook this eventuality. Samuelson reasons as a logician when he places the border between zero and non-zero. We believe that the scientist, who describes what humans do or should do, should place the border between the negligible and the non-negligible.

Finally, in the definition we propose, three points are to be highlighted.

(i) The first point is the "uniqueness" of realization defining a situation as random as opposed to heterogeneous: the definition of the type of a situation is totally *contingent upon the observer*. Therefore, this concept cannot be assimilated into the distinction between insurable risk and uninsurable risk, which is an intrinsic feature of the event, indifferent to the decision-maker. The purpose of our analysis is precisely to draw conclusions in terms of the relevance of descriptive tools applicable to the phenomenon *as regards the observer*.

Incorporating the decision-maker's subjective assessment into the definition of a situation might appear to be inappropriate. We do not consider this to be the case, as (i) even in polar cases, the nature of the situation as random or heterogeneous is already contingent on the position of the observer.; (ii) the notion of subjective probability, in both psychological and epistemological senses, is widely accepted; and (iii) we position ourselves in a framework dealing with decision-making and as such the relevance of the tool is inevitably intertwined with the decision-maker's appreciation of said situation (cf. the glacier example).

(ii) The second point is that the distinction between a random situation and a heterogeneous situation is based on the number of *future* realizations of the phenomenon. The precise knowledge of the past does not matter *a priori* in the distinction that we make here between random situations and heterogeneous situations. As a corollary, the distinction between risk and uncertainty (Knight, 1921) is not fundamental in our view. Indeed, random situations of risk and uncertainty assume common characteristics, just as do heterogeneous situations of risk and uncertainty; however, after having presented these transversal characteristics, we will underscore and make use of the *de facto* frequent proximity of random situations and uncertain situations.

(iii) Finally, in our view, as regards the pertinence of statistical indicators to the basis of a reasoning, the often highlighted opposition between thin-tailed and fat-tailed distributions (e.g. Taleb, 2007) is secondary to the distinction between randomness and heterogeneity. Of course, this affects the number of realizations that mark the threshold between a random situation and a heterogeneous situation, but as we will see, the difference of analysis required to manage these situations has to do with the difference in the nature of these situations and not with the thickness of the distribution tail.

1.3 Problematic

It is possible to mathematically define a distribution function in general, independent of the observer (molecule vs. experimenter, lottery ticket buyer vs. seller). To determine whether this distribution function – despite its mathematical uniqueness – takes on a different signification according to the observer's viewpoint, let us categorize the distribution functions of a random phenomenon according to the observer's situation (see below).

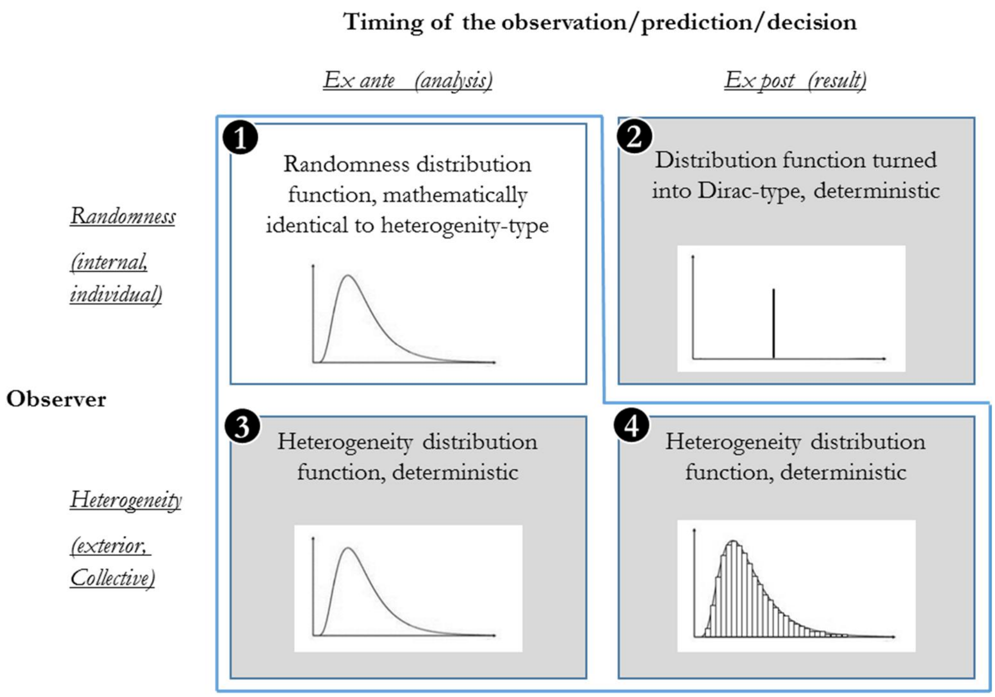

Figure 1: distribution functions according to situation

In figure 1, the first row corresponds to an observer placed in a random situation, and the second to an observer in a heterogeneous situation. The first column precedes the realization of the phenomenon (for example, one or several rolls of a die) and thus represents the distribution function of an individual phenomenon viewed by the observer *ex ante*; the second column corresponds to the view after the realization of the random event(s), and thus represents the distribution function noticed by the observer *ex post* (i.e., after the dice throws).

From a mathematical point of view, the distribution functions 1, 3, and 4, framed by a blue line, are identical: they correspond to the same equation – for example, a log-normal distribution (our die here has several faces and is biased). 1 and 3 correspond to the equation in a trivial way, as the probability distribution function of a dice throw is independent of the number of times we throw the die; 4 does so because if we throw the die a very large number of times, the experimentally observed distribution function will correspond to the probability distribution function. The distribution function for 2, however, is different: after having thrown the die a single time, we observe a single realization, and so the mathematical equation of the observed distribution function will be a Dirac delta function rather than a log-normal.

It is a different matter from an epistemic and operational point of view, because the distribution functions 2, 3, and 4, on a gray background, are identical in being equally deterministic. 2 and 4 are deterministic in a trivial way because they correspond to realizations. 3 is deterministic because we know that, over a very large number of dice throws, the entirety of observed results will form the corresponding distribution (4). In other words, functions 2, 3, and 4 are all descriptions of the result with which our observer is or will be faced. In contrast, the distribution function of 1 does not correspond to a description of an *actual* outcome.

The difference in nature between a random situation and a heterogeneous situation appears clearly in the way that the phenomenon's distribution function, in the case of heterogeneity, remains stable in 3 and 4, whereas in the case of randomness it transforms from 1 to 2. One situation is deterministic and the other is random. 3 is a prediction, whereas 1 is not. The statistical quantities associated with 3 are estimators of indicators of dispersion that will actually be observed, which is not true of 1.

Since functions 2 and 4 are different after the phenomenon's realization, one might naïvely wonder whether it would be justifiable to assimilate functions 1 and 3. This is precisely the question we ask:

- From a theoretical perspective, at the conceptual level: is function 1 an apt representation of the phenomenon to which our observer is confronted in a random situation, therefore providing insights which improve the quality of decision-making?

- From an operational perspective: does a confusion between these situations generate unfounded expectations and a biased representation, and is it thus a source of misinterpretation that negatively affects the quality of decision-making?

We will argue that it is not appropriate to use a type 1 function *ex ante* in an attempt to understand what will become a type 2 function *ex post* and to base a rational decision on it.

1.4 Results

<u>Absence of falsifiability and of responsibility</u>

In a heterogeneous situation (which is to say, when we are faced with function 3), the situation is deterministic and will transform into function 4, and we can therefore define the modeling error by measuring the gap between the two. In a random situation, however, when we are faced with function 1, having the function 1 *ex ante* authorizes any function 2 *ex post*: therefore there is no way to observe and measure the potential modeling error. Similarly, the distribution function can be back-tested by its observer in heterogeneous situations, but not in random situations, where the observer will always be faced, *ex post*, with a Dirac-type distribution function. Consequently,

Result 1: *in a random situation, there cannot be:*

- *from a scientific viewpoint, falsifiability of the model retained* ex ante,
- *from an operational viewpoint, accountability on the part of the modeler vis-à-vis the decision-maker in the case of an order of magnitude error for the model retained* ex ante.

Moreover, as regards risk models, the notion of *model error* – most often referred to as *model risk* – has been the subject of great interest since the crisis of 2008. Yet the result above can be reformulated as follows: a model of randomness is totally indissociable from the associated model error (they are fully fungible, while there is no porosity between a heterogeneous model and the associated model error). As such, risk models – indistinguishable from their associated model risks – do not exist in a random situation:

Corollary 1: *while the concept of the* dispersion model *makes sense in a* heterogeneous situation*, the very concept of a* risk model *is not pertinent in a* random situation.

<u>Non-pertinence of statistical indicators</u>

In a random situation, as one will be faced with only a single realization and as a type 1 function allows for any type 2 function *ex post,* a statistical indicator is not an estimator of any quantity to which one will be confronted *ex post*. One may wonder, then, as to its operational pertinence *ex ante*. What, for example, is the contribution to decision-making of an expected value that will never embody itself in an average?

It doesn't intuitively seem possible[3] to visualize an expected value or to explain it to someone without relying on the notion of an average. Even in a thought experiment, we cannot imagine the concept of expected value otherwise than as embodied in an average, just as, mathematically, expected value can be described only by a sum, be it continuous or discontinuous. Thus there appears a considerable difference between a heterogeneous situation and a random situation when we seek to explain this idea of expected value intuitively. In a heterogeneous situation, this idea may be explained *without* using the word 'if', as "the expected cost is the total cost divided by the number of policyholders". In a random situation, however, it becomes necessary to include an 'if' in the description, such as, "if one were to repeat the event identically several times". We thus introduce a condition that, in the case of a random situation, is impossible to fulfill because, by definition, an observer in a random situation is confronted with a single effective realization of the phenomenon. Consequently,

Result 2: *in a random situation, all reasoning based on an expected value or any other statistical quantity is flawed because it is based on the implicit reasoning that follows:*

Let A be the future repetition of the realization,
Let B be an apprehension of the underlying phenomenon and the resulting decision,

If A, then B
Not A…, then B.

Here, B is the direct consequence of the worldview A – a worldview which is an artifact that is the very foundation of the decision B. To consider a metaphor, this fallacious reasoning is of the same nature as if I said "*If* I lived in the 19th century, *then* I would not have running water and I would not shower. I live in the 21th century, *then* I will not shower".

To put it another way, this reasoning consists in considering that when we use a mathematical tool (a distribution function in this case), once the formalism of notation can be applied, the concept's domain of definition (the set of repeatable phenomena in this case) does not have to be taken into account. To consider a mathematical metaphor, the flaw in this reasoning is of the same nature, then, as that of someone who (i) observes that the antiderivative of $-1/x^2$, defined on R^*,

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3 This is a simple conjecture, but it is one for which, after having questioned many people about it, I am still not able to find any counter example.

is 1/x, and that therefore for all ordered pairs {a, b} whose elements are non-zero and of the same sign, the integral of this first function between a and b equals 1/b - 1/a, then (ii) deduces that for every ordered pair {a, b} whose elements are non-zero and of different signs, the integral of this first function between a and b equals 1/b - 1/a, because 1/b - 1/a exists mathematically when a and b are non-zero and of different signs.

This analysis can of course be generalized to statistical indicators other than expected value. For example: for a quantile, a heterogeneous situation allows for one to affirm that "when I make ten random draws, the worst is…", while relying on such a statistical indicator in a random situation forces us to posit a condition that is not and cannot be verified ("*if* I were to make 200 random draws, the worst would be…"), thus ruining the operational pertinence of what follows.

Let us point out here that the statistical indicators' pertinence for the basis of a reasoning rests on an opposition between the random or heterogeneous character of the situation, not on an opposition between thin-tailed and fat-tailed observed or putative distribution. On the level of a country, for example, even if income distribution is fat-tailed, the expected value of individual income has meaning (e.g., for the assessment of tax revenues for a flat tax, since the state is in a heterogeneous situation). By contrast, if I bet all my wealth double or nothing, expected value has no meaning even though this is a thin-tail distribution function. An expected value thus has meaning in a heterogeneous situation, even when faced with a fat-tail, and has no meaning in a random situation, even when faced with a thin-tail.

<u>Biased apprehension of the situation</u>

Bergson (1938, p. 156) criticized the way the metaphysicians and Zeno (trying to catch up the turtle) grasped change, explaining that "*what was considered as movement is neither change nor movement. They kept from movement what that does not move and from change what that does not change […]. Generally, we do look at change, but we do not see it. We speak about change, but we do not think about it. We say that change exists […] but these are only words, and we reason as if change did not exist*" As a consequence, Bergson calls for "*taking away the artificial schemes that we interpose obliviously between the reality and us*". This analysis could be straightforwardly transposed from change and movement to risk and randomness. Indeed, the use of statistical tools corrupts the observer's apprehension of the situation, inducing a perception of the risk models as predictive – and generating afterwards disappointments against these risk models "that hadn't foreseen that…".

When making a decision with the help of a type 1 function, the intuition we call on to understand the tool we are using rests on the assimilation of type 1 into type 3. In a heterogeneous situation (type 3), we can effectively *predict* because, with a certain margin of error, the phenomenon is (overall, statistically) deterministic. In a random situation, the decision-maker using a type 1 description of the situation is thus led by this "frequency" metaphor to use a system of thought where the phenomenon is considered to be repeatable and to forget that statistical do not exist in the decision-maker's world. *The expected value is thus wrongly perceived as an estimator of the result*, as if the actual result would be an average and not a unique outcome, as if he were going to be met with a type 4 distribution *ex post*, rather than a type 2. And this without the actors remembering that *the very foundation of a choice of apprehending a phenomenon as random is the fact that the width of outcome is too large for a central anticipation (e.g. expected value) to make sense.*

Such fallacious perception can be illustrated in the following exchange, observed in fieldwork during a large financial company's investment committee meeting:

- CFO: *"we have a higher level of risk than the market [than our competitors]: our equity allocation is higher, our real estate allocation is higher…"*
- A critical participant: "*in that case, we should have a higher rate of return. I don't think that we're really above it*".

The expected returns – higher than the market standards in the risk/return framework chosen by the company – is seen here as a estimator of what should have been obtained. The participant thus considers it abnormal (i.e., unnatural, a reason for suspicion, a hint of an error or a lie) that the observed returns are lower.

This impact on feelings is natural. Indeed, the process of quantifying requires a change of perspective: to quantify, one must place oneself in a position from which the observed phenomenon becomes deterministic (from the policyholder to the insurer, for example, where 3,000 in 100,000 policyholders will develop a cancer). This gives us the feeling that we *know* what is going to happen[4]. And it would be true from this new point of view… but this is not the decision-maker's perspective – the decision-maker *does not know*, and this is the very thing that constitutes the risk. To state it formally,

[4] Take for example a simple coin toss: it is not at all the same thing to say that the result is 0 *or* 1, *without being able to know* which will be the case before observing the result and to say that the result will have an expected value of 0.5 *with* a standard deviation of 0.5, which we *know* to be true (and thus certain).

Result 3: *The use of statistical indicators leads to a distortion of our apprehension of the world, because these indicators are based on the construction of a system of thought that distances us from reality. By assimilating random situations into heterogeneous situations, it leads to a deterministic perception of phenomena, which are reduced to a set of indicators whose value is characterized* ex ante*.*

The obliviousness of users as to the change in the nature of their models explains how certain CxOs who steer major financial actors are convinced that their Basel II or Solvency II risk models are "predictive" (and maintain their position when opposed), even though this is contradictory with the very concept of "risk model". As a direct consequence, they base their decisions on erroneous mental representations, where the risk has disappeared *de facto* from the decision-maker's field of perception.[5]

Additional result

To estimate the expected value or any other statistics, one first has to define the sample. Calibration attempts will then encounter the reference class problem, which was already discussed at length in the 1930s (McGoun, 1995), and has since been recalled from Reichenbach (1940) to Eagle (2004).

In practice, when the number of future realizations of a phenomenon is low for a given observer, the total number of observations (both past and future) for this observer is generally low. This is either due to the total number of realizations, regardless of the observer, being low or because, on the contrary, the phenomenon in question can occur in a heterogeneous situation, *but for other observers*[6]. As a consequence, the reference class issue is more intense: as von

[5] More broadly, this kind of bias in the representations we make of the world leads to making decisions related to randomness on the basis of a cost-effectiveness analysis, such as when signing a reinsurance treaty, a CFO or CRO compares its price to a deterministic "cost of risk", or when an investor considers the Sharpe ratio or one of its embodiments. This kind of practice marks the final stage in the process of drifting from the apprehension of risks to a quantified risk/return couple, where risk is completely absorbed into a single synthetic performance indicator. Risk has thus completely disappeared from the decision-maker's perception of his environment, leaving him with a control panel that has been reduced to one figure and stripped of all randomness.

[6] Knight (1921, pp. 224-225) distinguishes three types of situations: *a priori* probabilities, which are purely mathematical and correspond to a dice throw, for example: statistical probabilities, which concern the evaluation of empirical frequency within a class of events considered to be homogeneous; and estimates, for which there is "*no valid basis of any kind* for classifying instances" [his italics]. In accordance with our analysis above, we consider that the distinction between these last two categories does not depend on the intrinsic nature of the phenomenon but rather on

Mises (1949) pointed out, a unique event is never a member of a class, and therefore, in a practical random situation[7], quantification does not allow to realize the practitioners' usual ambition of objectivity.

1.5 Summary of the theoretical framework

We have laid out the theoretical framework that allows us to distinguish two kinds of different phenomenon: random situations and heterogeneous situations.

(1) When a decision-maker is confronted with a random event that will occur only once in the context of his decision-making, he is in a random situation. If, within the context of his decision-making, this event will occur an infinite number of times, he is in a heterogeneous situation.
(2) In heterogeneous situations, the decision-maker has a deterministic understanding of the world: he knows *ex ante* the distribution function of the events he will observe *ex post*. This is not the case in random situations.
(3) This alternatively predictable or unpredictable character of the result is what allows us, more generally, to demarcate heterogeneous phenomena from random phenomena in non-polar cases where the number of future realizations is more than 1 but is not infinite: if the forecast error for the distribution function observed *ex post* by the decision-maker is negligible in his eyes, it is a heterogeneous situation. If not, it is a random situation.

In random situations, the reliance on statistical quantities is based on a conflation of randomness and heterogeneity. The principle consequences of this are as follows:

In a random situation,

(1) Risk model and model risk become indistinguishable; it is impossible to determine accountability regarding the model's quality *ex post*;
(2) To base a decision on the statistical quantities associated with the distribution function expected *ex ante* relies on a fallacious reasoning;
(3) Additionally, such an analysis alters the decision-maker's perception of the phenomenon, giving him an illusion of determinism, skewing his reasoning, and impairing his ability to an appropriate decision.

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the observer's position relative to the phenomenon. This will appear very explicitly in the example concerning insurance contracts in the second section of this paper.

[7] By this expression, I mean a random situation whose underlying phenomenon belongs to neither the category of "quantum mechanics" nor of "games of chance".

Finally, it is impossible to calibrate an *ex ante* distribution function without arbitrariness. This negates the claims to objectivity of all quantified risk models.

2. Typology of the fields of application

"Not only is it not right, it isn't even wrong."
W. Pauli

Either on its own or because it highlights common confusions, the distinction between randomness and heterogeneity offers a broad range of applications. Below, we will establish a typology of these fields and present or allude to illustrative examples from each of them.

The first field is that of *solipsistic decision*, where an individual is confronted with a choice in a random situation and where we tend to use descriptive and normative tools to characterize decision-making which come from a heterogeneous perspective. We will consider two examples: decision theory and investment.

The second field is that of *asymmetrical interactions*, where individuals in random situations and individuals in heterogeneous situations coexist in the context of the same underlying random phenomenon. We then have a tendency to tack a methodology suitable to the situation of one actor onto that of the other. We will present two examples of this field: public choice, where the outlook of society is sometimes apprehended as an aggregate of individual outlooks, and rate differentiation in insurance, where, conversely, the insurer's perspective is perceived as legitimate to describe the policyholder's situation.

Finally, the third field is that of *third-party regulation*, particularly in the financial sector, where a regulator intervenes in the relationship between financial institution and client. The distinction between randomness and heterogeneity here is especially rich and can have serious consequences in the design of prudential policies. For our illustrations, we will evoke different aspects of the Solvency II reform, which regulates the European insurance industry.

2.1 Solipsistic decision

Decision theory

Many decisions are made in random situations. Of course, an individual may face a succession of events over the course of his lifetime, but we cannot consider this set of events as constitutive of a heterogeneous situation because the events are not independent of one another. The decisions concerning one's job, romantic life, investments, health, and so on, all interact with one another, just like how in a company, choices regarding IT investment, product strategy, and business development all interact. As such, an individual's or a company's life cannot be seen as a succession of independent lotteries and significant decisions under uncertainty are thus made in random situations. However, decision theory revolves around a statistical quantity – the expected utility – either by way of the classical formalization by von Neumann and Morgenstern (1944) or one of its avatars, or by way of its axiomatization by Savage (1954).

The beginnings of decision theory were accompanied by muffled doubts concerning the relevance of the concepts (Allais, 1953)[8] and the tool has since come under greater criticism. In response to the descriptive criticisms of the initial proposal, economists adapted the model to grasp what were considered as cognitive or psychological biases – as with Friedman and Savage (1948) introducing a concave utility above income and a convex one below it to describe the simultaneous purchase of insurance and participation in a lottery; or like Khaneman and Tversky (1979) similarly introducing a utility function of initial wealth to reflect some of their empirical observations; or, more generally, all attempts to distort either the utility function or the probability function with subjective probabilities; or, lastly, as numerous models that were built which, albeit belonging to the theoretical field called "*non-expected utility*", rely heavily on expected utility, e.g. *rank dependent expected utility*, *maximin expected utility*, or *Choquet expected utility* –. Decision theorists have since come to the conclusion that no model built on these bases can successfully combine the necessary answers to the different observations in one global theory: Camerer (1989) is particularly pessimistic, remarking that this would be too complicated, that a list of all the observed breaches would be too long, and that identifying the implications of the different theories on the results obtained through the expected utility would be difficult. More recently Barberis (2013), while highlighting the last decade's contributions to the psychology of tail events, explains that two problems remain: in the analytic (intellectual) phase, the overestimation of rare events is sometimes an underestimation, and in the decision (psychological) phase, the overweighting of associated probabilities is sometimes an underestimation; in other words, nothing emerges clearly when we consider expected value as referential. From a normative viewpoint, Gilboa (2009) demonstrated that Savage's axioms do not correspond as such to rational decision criteria. However, expected utility – or one of its many embodiments – still provides the

[8] Although Allais places "*the consideration of expected value*" among the "*four [fundamental] elements that all theories of risk must* necessarily *take into account*", he mentions in passing that it "is obvious that, *if it is a question of a single dice throw that will not recur, the justification founded on the law of large numbers does not apply at all.* However, and precisely, *most cases where we have to make random decisions are isolated cases*".

frame of reference against which we (i) judge the intellectual pertinence of choices (whether or not they respect the expected utility) and (ii) gauge the psychological preferences of decision-makers (risk premium defined as the difference between expected utility and utility of expectation, e.g., Pratt, 1964).

Our present analysis offers an understanding of why the quest of such a theory is doomed to failure: when the decision-maker is in a random situation, expected value (or any other aggregate statistical indicator), being empty of operational meaning, cannot be anything other than an arbitrary frame of reference for the decision-maker. The arbitrary nature of this frame of reference can explain the long march toward the *ad hoc* made by a decision theory that now seems without perspective, in the same way that a geocentric frame of reference, due to its arbitrariness, condemned astronomers to put *ad hoc* mechanisms into place to describe the movement of planets before finally abandoning them.

This observation has major impacts. (i) EU has no normative significance. (ii) Once expected value is disqualified as meaningless, risk aversion, constructed around the notion of expected value, no longer has meaning since its point of reference has none. This is a positive step because this kind of abusive interpretation led utility function into impasses such as the paradox of people who are simultaneously risk averse (seek insurance) and riskophilic (playing a lottery). Such an observation is no longer paradoxical as soon as we let go of expected value as our point of reference in building the concept of risk aversion. In our framework, a unique mathematical property (concavity of the VNM utility function or being above its chords) is no longer employed to explain two orthogonal characteristics (both satisfaction and a fallacious "risk aversion"). (iii) Other impacts have to be further explored, both on empirical issues (e.g., relevancy of low stake experiments, cf. conclusion) and on theoretical issues (e.g., the possibility of explaining aversion to ambiguity in our framework[9]), some of them being listed in our conclusion.

We must recognize that the danger of our critique is that is doesn't permit the substitution of another statistical quantity for expected value: there is no alternative quantitative statistical frame of reference that overcomes the critiques we have addressed to expected utility. In the final section, we will propose an

[9] Aversion to ambiguity is generally tested in the form of a bet, placing the test subject in the environment of a repeated game; in such a situation, the risk is perceived as heterogeneous, while the uncertainty is still perceived as random because, since the subject does not know the initial probability, he cannot exclude that the probability of an adverse result is not negligible (even after a significant number of repetitions). Therefore, the problem of ambiguity is not a ignorance of probabilities but rather of the range of non-negligible possibilities.

alternative methodology that does not rely on a statistic indicator and which offers a greater relevance – at the expense of a significant loss in tractability.

Investment

Asset allocation decisions are generally objectivized by basing them on the comparison of risk/return profiles of different conceivable portfolios. However, decisions regarding strategic asset allocation are only made a few times over the course of an executive's career, and the decision-maker cannot pool returns over a long period (several decades) when he is accountable to his shareholders and his clients. The same applies to a private individual investing over a given period, be it for several years. Investors are thus in a random situation.

Subsequently, the issue of the pertinence of a decision based on statistical indicators such as a risk/return couple arises. This issue is easy to grasp intuitively: either the risk is considered to be non-negligible and the expected value, which is not an estimator of the result that will be obtained, is not an evaluation of what will *effectively* be observed; or the risk is considered to be negligible, in which case it is no longer useful to take a risk/return ratio into consideration. In other words, the use of statistical tools is based on a temporal inconsistency: in the short-run, the risk is to be considered, but the expected value is meaningless; in the long-run, the expected value becomes a tangible measure but the risk disappears. A risk/return couple thus offers to the decision maker a set of indicators which is internally inconsistent.[10]

2.2 Asymmetrical interactions

Individuals and society

Clarifying the confusion between random situations and heterogeneous situations is fruitful in many fields related to public decisions. In general, the passage from the collective (heterogeneous) to the individual (random) and vice-versa requires a change of perspective, though this necessity is not always perceived. Conversely, this distinction helps to explain certain juridical mechanisms as heuristics that enable us to process the necessary difference in approach.

In his article on the *Fallacy of Large Numbers*, Samuelson (op. cit.) pursues his argument by breaking down a heterogeneous situation, representing the associated overall decision as a succession of decisions made, one after another,

[10] This problematic will be the central subject of a future article detailing the discussion, providing quantified examples and also exploring the lack of falsifiability issue and the consequences regarding governance and responsibility.

in random situations. Our position, consistent with Martin and Pindyck (2015)[11] is that it is not apposite to do so. For example: in the sixties, when the French government launched Airbus, Concorde, the TGV trains, Plan Calcul (an attempt to compete with IBM), Ariane, and civil nuclear power, or now when Google simultaneously gets involved in genomics, connected glasses, and self-driving cars, these sets of programs are comprehensive policies that should be understood in their entirety. *Ex post*, we observe the pertinence (or non-pertinence) of these comprehensive policies as *one* decision – this decision forms an indivisible whole, and we cannot consider that certain sub-decisions should have been made and others not *ex ante*. It would not make sense to blame the French government for having tried to launch the Plan Calcul or Google for having invested in Google Glass because, had they not, there would not have been investments in Airbus, civil nuclear power, and self-driving cars, either.

Likewise, the fact that a choice can be pertinent in a heterogeneous situation and not in a random situation may provide some explanation for the fact that, in the field of evolution, organisms undergo random genetic changes. Of course, randomness is certainly not desirable on the individual level, as mutations are often imperfections: an individual given the opportunity to have a randomly deviant gene would certainly refuse. But the heterogeneity of these mutations is beneficial to the species, which can allow for punctual losses and stands to gain a great deal in the event of an unexpected development of new capacities: expected value, which makes no sense for the individual, does make sense for the species.

A risk that is too high for an individual can thus be constitutive of a heterogeneity that is beneficial to society, and vice versa. Faced with these differences of pertinence between an individual choice in a random situation and the accumulation of individual choices in a society that is in a heterogeneous situation, our societies seem to have developed heuristics to distort individual decision-making through mechanisms that alternatively curb or promote risk taking. In the first category, we find examples such as prudential regulations, which aim to prevent financial institutions and their executives from taking as many risks as they like so as to limit bankruptcy. In contrast, the second category includes examples such as limited liability companies, which protect executives and shareholders from being held responsible for unlimited losses and protect them from the claims of creditors in the case of bankruptcy. What is the difference between these two categories? In the first case, it is the savers who

[11] Though they do not distinguish randomness from heterogeneity, they show that, when faced with the possibility of avoiding a set of random events, sequential reasoning is not optimal: "*although naïve reasoning would suggest using a sequential decision […], such a rule is not optimal*".

suffer the consequences of bankruptcy, and in the second, it is the suppliers. Savers generally do not open several retirement or savings accounts – and anyway the financial system is exposed to systemic risk: they are in a random situation regarding the possible bankruptcy of their bankers. The issue is thus no longer expected value: the possibility of ruin would be too painful for the savers, so it is necessary to limit the potential and the impact of such an eventuality. By limiting risk taking that would be optimal for the shareholders of the regulated company (and who diversify their investments, thereby placing themselves in a heterogeneous situation that may push for each company to take risks), society protects the savers, who are in a random situation. Conversely, if a standard non-financial company goes bankrupt, it is said company's suppliers who will not be reimbursed and will lose a part of their revenue. Most suppliers, however, are in a heterogeneous situation: statistically, they already know that they will suffer losses due to one client or another, and a gains/losses balance can then be established socially. Such a balance is effective and must not be disturbed by an executive, who would be in a random situation if he had to make up for the bankruptcy with his own assets. Society limits this potential disturbance of social efficiency by limiting the small business executive's responsibility. Such comparisons raise the question of whether or not it is appropriate to maintain a limited liability for shareholders and top managers of financial institutions.

This distinction between an individual faced with randomness and a society faced with heterogeneity can be applied on a higher level – that of society as the human species. As our species has acquired the capacity to alter the planet in its entirety (climate, GMOs, etc.), it is humanity as a whole that finds itself in a random situation as regards certain choices. Taleb et al. (2014) indicate that the precautionary principle should be applied to GMOs and not to nuclear power, as they base their argument on the fat-tail/thin-tail distinction and the risk/uncertainty distinction. We agree with their conclusion, but it could be reached more simply and more directly with our dichotomy. As such, GMOs have worldwide impact and represent a random situation for humankind: while they cannot be analyzed in terms of risk/return, this is not so much because the probabilities are poorly known, as Taleb et al. argue, but rather because in such a case a decision cannot be made on the basis of statistical indicators. Conversely, on the scale of a country with several dozen nuclear power plants operating over several decades, an accident in one of these plants remains within the framework of heterogeneity for society and can be analyzed through a risk/return analysis.

Insurance rates and differentiation

Another field of asymmetric interaction is that of pricing in insurance and the relationship between the policyholder and the insurer. Rates have historically corresponded to the "price of risk" of a rate class (which is rather large), to which a margin is added. With progress in genomics, the rise of social networks, the arrival of connected objects, the development of information technology, and advances in the data sciences, big data enables increasingly finer segmentation: from a technical perspective, this refines the pure premium according to individual risk, and from a commercial point of view, it refines the margin according to the individual willingness to pay. Faced with this change in the landscape, European and American societies, driven by the same desire to avoid differentiation considered undesirable (undue discrimination), have reacted in opposite ways. Europe limits possible differentiations of the pure premium, prohibiting insurers, for example, from basing motor insurance or pension rates on the policyholder's sex (European Council, 2004; CJEU, 2011). In its view, fairness leads to limiting the trend of pricing at "the price of one's risk". Conversely, the Consumer Federation of America (CFA, 2013) seeks to prohibit price optimization – the fixing of margins based on the willingness to pay – stressing that "*moving pricing away from its historic cost-based approach*" is "*unfairly discriminatory*". This position follows a tradition that judges the pure premium to be the frame of reference for determining the "fair price" (Daston, 1989; Feller, 1968, p. 249). One might ask, then, is the pure premium the "fair price" for the policyholder?

While the insurer is in a heterogeneous situation, the policyholder is in a random situation. Expected value is a pertinent tool for the insurer, for whom it is embodied in an average (if the models are good); but there is no reason to transpose this reasoning onto the policyholder's situation by considering that the cost for the producer would correspond to the "fair price" for the consumer. The expected value is nothing but the cost for the insurer, and in many markets the cost that a customer is ready to pay is not the production cost. For luxury goods, it is much higher; for subsidized products (e.g. railway or farming products), it may be much lower. As expected value is meaningless in a random situation, the pure premium has no special legitimacy in the eyes of the policyholder and, *from his point of view*, such postulated "fair price" does not correspond to a criterion of justice. A policyholder's demand to "pay the price of his risk" thus appears unfounded: imposing limits on rate differentiation as is done in Europe cannot be contested *in the name of an individual ethics*, and promoting a rate based in principle on the cost of risk as being "fair", as certain American associations do, is moot. This observation leads to refocus the debate over pricing discrimination on the question of its economic efficiency (balance between exclusion and moral hazard), with the right price being nothing other than the competitive price on the regulated market (i.e. market with potential technical differentiation prohibition).

2.3 Third-party regulation

For the past fifteen years, the European Union has been implementing a new prudential regulation in the insurance sector (European Commission, 2009) that, in its quantitative dimension, is based on capital requirements corresponding to an annual VaR of $1/200$. In this context, the distinction between randomness and heterogeneity has profound ramifications[12]; below, we will mention just two.

1. Legislators and regulators are faced with an industry composed of numerous companies and are in a heterogeneous situation. As a consequence, their calibration of the capital requirement may make sense: it can be challenged and they can be held accountable if a crisis reveals it to be inappropriate. But the regulation authorizes companies to use internal models to determine the $1/200$ VaR while the company itself is in a random situation in regard to its ability to survive. Due to the porosity between the risk model and the model risk and the resulting lack of accountability in such a situation, one cannot scientifically objectivize *ex ante* nor *ex post* the pertinence of those internal models.

2. The regulation mentions a calibration of capital requirements to 99.5% VaR, which seems to be mathematically univocal. But how should this sum be calibrated? Should we look at the heterogeneity of the companies that make up the entire industry, in which case we would expect approximately one of every 200 companies to go bankrupt each year (option 1) and the calibrations to be reviewed annually, or should we look at the heterogeneity of the situation of financial markets in time, in which case the calibration, company by company, should remain stable over time and be such that each company statistically goes bankrupt one year out of every 200 years or so (option 2)? As the regulation does not distinguish between randomness and heterogeneity, the targeted scope of heterogeneity has never been defined: it has not been taken into account that it should have specified whether the operational interpretation of its quantile should be temporal (option 2) or geographical (option 1). Mathematics of randomness and of heterogeneity are the same, so the regulatory quantum seemed univocally defined, but as

[12] These problematics will be the central subject of a future article.

randomness and heterogeneity are distinct issues, the interpretation is equivocal and the calibration remains undefined.[13]

More broadly, the fact that financial institutions and their regulators tended to rely excessively on quantitative models before the 2008 crisis (Ashby, 2011) can be explained by the fact that they were accustomed to using successful models in a business/pricing context. When these models were transposed onto a steering/risks context, the nature of the models changed – they transformed from predictive models (heterogeneity) to non-predictive models (randomness) – and this change in their nature has not been perceived by their users nor by their environment. Overall, this raises the question of the pertinence of risk management as adopted by CROs that Mikes (2011) classifies as "quantitative enthusiasts".

3. What alternative is there?

"The supreme exercise of intelligence is decision. […]. The decision, once we have performed 80% of the analysis, is the chasm between the moment you jump and the moment you succeed. So it requires courage, because you must jump. There is no intelligence without courage."

O. de Kersauson, *Le Monde comme il me parle*

3.1 The problem to be solved

The main problems arising from the use of statistical quantities to apprehend randomness are the following:

(i) They distort the decision-maker's perception of the situation because as soon as the risk is quantified, it is no longer perceived as a risk. This distorted perception skews the decision, which can no longer be in line with the decision-maker's situation, and thus cannot be considered rational on his part[14].

(ii) The decision is made on quantitative foundations that are both non-objective and technically complex, so the underlying choices are not accessible to decision-makers. In other words, the subjectivity – that is supposed to be the prerogative of the decision-maker – is

[13] Of course, public authorities have an interest in the result being that "0.5% of the market goes bankrupt each year", which they would find manageable, but this hardly seems to be their position, as the calibrations are not revised annually.

[14] Blamont (2005) observes that "science brings itself into being in opposition to the witness of our senses, […] in opposition to intuition". Indeed, science may have to proceed this way because to be objective is to be without a point of view, but the decision-maker is not an analyst: he *must* have a point of view: his decision is (rationally) contingent on his position.

delegated to the analyst, and thus the decision is not made at the right level.
Moreover, given that (a) the decision-maker is considered rational if and only if he uses the results from these calibrations, and (b) the quality of the calibration provided by the analyst can never be evaluated, it is no longer possible to establish accountability for the quality of the decision-making.

In order to avoid these flaws, the tools on which to base a decision under uncertainty must allow for (i) *the feeling of randomness to be reintroduced* and for (ii) *subjectivity to be assumed at the correct level*.

Renouncing statistics requires that we renounce (i) the ambition to compact the potential outcomes to the point of reducing them to one or several deterministic statistical indicators that claim to synthesize all possibilities and (ii) the ambition to attain quantitative objectivity, which seems an impossible goal in our view.

3.2 The structure of a solution

According to Solé (2000), individuals make their decisions by categorizing future scenarios into three classes: that which cannot occur (Impossibles), that which could occur (Possibles), and that which is inevitable (Non-Impossibles, which we cannot imagine not occurring). These mental schemas are largely the unconscious result of each individual's culture and experience. They define our perception of reality: a mental schema of analysis that frames decision-making for each of us. We propose to start from this decision-making schema, amended to account for the fact that the decision-maker is faced with a consciously random situation: he knows that he does not know and formalizes his decision in consideration of this dimension of uncertainty.

Let us first propose an intuition of the process of analysis and decision. It is based on the following steps:

(i) Identification of all possible states of nature: Opening the decision-maker's field of potential Possibles.
In the case of an asset allocation decision, for example, a typology of possible scenarios could be: rise of the spread of government bonds accompanied by a drop in stock prices and a rise in property prices; negative rates accompanied by a rise in stock and property prices; etc.

(ii) After the most exhaustive inventory possible: The rejection of those scenarios that would be considered "Impossible" *by the decision-maker* so as to retain only *his* "Possibles".

By "Impossibles", we mean those scenarios for which the decision-maker does not want to take into account the possible realization either because he judges it to be negligible (literally Solé's Impossibles) or because he accepts to assume the risk (we here widen his acceptance). In the case of an asset allocation, for example, the decision-maker would have to take a position on whether or not he considers a default of his country's govies as an "Impossible".

(iii) The analysis of the impact of various possible actions in each of the remaining "Possible" scenarios: The choice of the action that maximizes the decision-maker's utility in the scenario in which he "believes" most, under the condition that the consequences are acceptable in each of the Possible cases. In our investment example, this would consist in choosing the allocation that maximizes the returns in the preferred scenario, under the condition that these returns remain acceptable in his other Possible scenarios.

Such a process may surprise because it does not make use of probabilities as such, only the parameter "negligible or not"; but from a normative perspective, it is consistent with the lack of meaning of statistical indicators (such as probabilities) in a random situation, and from a descriptive point of view, it seems a natural fit with Solé's analyses[15].

Let us propose a formalized version of this process:

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[15] Of course, this contradicts Arrow's apologue (1951), which uses the following example to justify the claim that a decision-making reasoning under risk cannot do without probabilities: "*If an individual were told to predict whether or not two heads would come up in successive throws of a fair coin and further informed that he would lose his life if he guessed wrong, I find it very hard to believe that he would disregard the evidence of the calculus of probability[…] [A]n extension of this suggests that in almost any reasonable view of probability theory the probability of a single event must still be the basis of action where there are genuine probabilities.*" But, beyond the fact that he ignores the central problem of reference class for a choice made in a more realistic context, this example overly simplifies another aspect of "real life".

A choice such as the one proposed by Arrow is between $-\infty$ (to die) with a probability p (1/4) or with probability 1-p (3/4). In a real situation, even a highly simplified one, an individual is not faced with the choice between A with probability p and A with probability 1-p. Let us suppose for example that for your retirement, you can put your 1 M$ savings into a lottery that will pay out 2 M$ if you win. This means that you have the choice between ultimately holding onto 1 M$ with the probability p=1 or playing for 2 M$ with the probability p' vs. 0 with probability 1-p'. If we ask the question "does the result depend on whether p' equals 1/4 or 3/4 ?", Arrow's claim becomes much less self-evident. The only issue at stake here, in deciding whether or not to make the bet, becomes: "can I ignore the possibility of losing my savings?" or, in other words, "does losing belong to what I subjectively consider an Impossible?".

Let S be the set of all existing states of nature,
of which S_i is the set of all states of nature that agent i imagines,
of which $\overline{S_\iota}$ is the set of all states of nature that agent i identifies as Possibles.

For example, the case of a giant meteorite razing Paris ∈ $\{S - S_i\}$; the case of a centenial Paris flood recurring this year ∈ $\overline{S_\iota}$ for the homeland security (which is prepared for such an eventuality), but ∈ $\{S_i - \overline{S_\iota}\}$ for the organizers of Roland-Garros, who certainly accept to disregard the possibility of such an occurrence, or, in other words, run the risk of being unprepared for a flood (and for which nobody would blame them).

Likewise, let A be the set of actions that a human being can imagine,
of which A_i is the set of actions that agent i imagines,
of which $\overline{A_\iota}$ is the set of actions that agent i identifies as Possibles.

For example, the action of killing one's parents to inherit their wealth faster certainly ∈ $\{A - A_i\}$ because the idea would not even occur to us; according to the person, the case of cheating on one's partner belongs either to $\overline{A_\iota}$, or to $\{A_i - \overline{A_\iota}\}$ if, as a matter of principle, *i* considers it taboo; if it belongs to $\overline{A_\iota}$, then the question of cheating – which is to say the consequences of the risk – will arise.

Now, let $\dot{S_\iota}$ be the state of nature in which agent i believes most

Let us note that U(a; s), the pay-off associated with the action a, combined with the state of nature s, and $\overline{U_\iota}$, the worst result that i finds acceptable.

Proposal 1 :

Thus, the decision $\dot{A_\iota}$ made by agent i is the decision such that its pay-off $\dot{U_\iota}$ satisfies:

$$\dot{U_\iota} = \begin{cases} \max_{\overline{A_\iota}}(U(a; \dot{S_\iota})) \\ under\ constraint\ \min_{\overline{S_\iota}}(U(\dot{A_\iota}, s)) \geq \overline{U_\iota} \end{cases}$$

Let us note here that it is possible to draw a certain number of parallels with some usual concepts of decision theory: the fact that A_i and S_i are only subsets of A and S corresponds to a Simon-type bounded rationality; the subjectivity associated with $\overline{A_\iota}$, $\overline{S_\iota}$, and $\dot{S_\iota}$ is linked to the fact that we consider subjective probabilities in expected utility; and finally, $\overline{U_\iota}$ can be understood as an equivalent to a parameter such as risk aversion.

There is, however, a fundamental difference between the two approaches. The *decision* here does not reside in the mechanical operation of constrained optimization as it is implicitly in decision theory. *What we highlight is the fact that the analysis is (the most important) part of the decision: the decision resides in the choice of $\overline{A_\iota}$, $\overline{S_\iota}$, $\dot{S_\iota}$ and $\overline{U_\iota}$ – a choice that is committed and conscious, and not a passive characteristic of the individual.*

This choice is irreducibly a-rational (and subjective).

3.3 Advantages / disadvantages

Disadvantages

From a theoretical viewpoint: A decision theory likely to be embedded as a step in a larger reasoning must *of necessity* rely on the ability to aggregate the set of a decision's outputs according to the different states of nature imaginable – that is to say, it must be able to establish a statistical indicator so as to compare its value in the context of different actions. The pertinence of such an indicator – whatever it be – in a random situation is precisely what we affirm to be impossible. In other words, the price of our descriptive and normative pertinence is a renunciation of a ready-to-embed decision theory. The process that we propose is, of course, an alternative to decision theory but, as game theory, *it is an alternative with a significantly downgraded ability to infer general results in the framework of a mathematical formalism that makes use of closed formulas,* and an alternative where the characteristic of the agent are contingent to the issue – for example, there is no general risk-aversion, but sets of $\overline{A_\iota}$ and $\overline{S_\iota}$. [16]

From an operational viewpoint: The chief flaw of this process is its lengthiness, as it can require multiple iterations and, potentially, a revision of the notion of "acceptable consequences"[17]. Furthermore, from a psychological point of view, this process is harder to bear for the decision-maker insofar as he must take

[16] However, while this process is less "tractable" than that of expected utility, it is no less "quantitative" nor less "objective", as methods based on an expected value always call on distortions of the probability function (and on utility functions) that in the end do not allow generalization (Barberis, op. cit.).

[17] Or even of the "Impossible" classification in the second step. In the theoretical outline, acceptable consequences and the classification of scenarios as Possible or Impossible are two hermetically disconnected and independent decisions. However, just as when one simulates a change or builds a strategy, there may be feedback from the objectives we set out for ourselves regarding the means at our disposal, with the latter not unilaterally determining the former according to an accounting logic, likewise back-and-forths can lead to a revision of the notion of "acceptable consequences".

responsibility for the arbitrary aspect of his decision and cannot shift blame onto a hidden technician or an ethereal statistic.

Advantages

While our methodology is more time-consuming for the decision-maker, this is because it obliges him to appropriate the analysis for himself, making it indissociable from the decision, as the categorization of an eventuality as Impossible (or, negatively, its non-identification as such) concerns both analysis and decision-making. This porosity seems in line with contributions from disciplines other than economics, which underscore an inseparability between analytics and "emotional" anticipation of the future in decision-making (Damasio, 1995). It is also coherent, in organizational terms, in regard to the mechanisms of analysis and decision-making observed in large institutions where there is a continuum between technical and political actors (expert, manager, director, executive committee).

Moreover, once expected value is disqualified as meaningless, we have seen that there is nothing rational about expected utility in random situations. The porosity between analysis and decision occasioned by this methodology allows us to extricate ourselves from a framework where divergences from the principle of expected value are seen as psychological/cognitive biases.

Furthermore, our process extricates us from an aporia. Gilboa et al. (2008) underscored a tautological impasse inherent in Savage's axiomatization, which requires the agent to have a preference order *ex ante* and which could therefore hardly help in making a decision. Under the Expected Utility formalization, we face, albeit in a less explicit way, the same blocking self-reference. For example, when I asked one economist what he meant when he maintained that "X had one chance in two to win the next election", he replied, "*when I said a 0.5 probability, I meant that is what I would use if I had to maximize my utility function*"[18] [19]. Our methodology allows us to get rid of such aporia, as $\overline{A_\iota}$ and $\overline{S_\iota}$ are not outputs of the results.

[18] It was in fact the only possible response to von Mises' critique: *"[to say for a unique event that 'there are 9 chances in 10' is] a metaphorical expression. […] There is no use in applying the yardstick of logic to a critique of metaphorical language. […] the comparison is based on a conception which is in itself faulty in the very frame of the calculus of probability, namely the gambler's fallacy. […] it is implied that this ratio [p] tells us something substantial about the outcome of the unique case in which we are interested. There is no need to repeat that this is a mistaken idea."*

[19] From a historical viewpoint, expected value was defined first and probabilities came afterwards, defined at their beginnings as the ratio between the expected value and the outcomes (see Daston, 1988).

3.4 A methodology coherent with certain heuristics

While this methodology is antagonistic with the current canons of decision theory, it is, however, in line with certain professional practices.

Some heuristics (currently heading toward extinction due to the growth of financial methods) are occasionally privileged over standard statistical outputs by professionals who consider that, as we have shown, a profitability analysis would not be pertinent. As such, one technique used in choosing a reinsurance treaty, for example, is building nomograms {number of serious claims, cost of these claims} and identifying the main areas that appear in order to determine how to cover for risk in accordance with how this corresponds to "scenarios" that seem qualitatively credible or not.

More generally, the same principle applies to regulation when a scenario-based approach is opposed to a statistical approach.

Finally, our approach corresponds to a focus on the possible consequences of our decisions, which also corresponds to Taleb's recommendations (2007), but for different reasons. As we mentioned in section 2, Taleb considers that we cannot know the probabilities of rare events; we judge that as soon as we decide to consider their occurrence as possible, their probability is not an issue. As such, in terms of asset allocation, a 1/N allocation strategy such as Taleb (op. cit.) or Haldane and Madouros (2012) recommend corresponds to not wanting to exclude from the field of Possibles any scenario in which a given asset's value falls. The old rules of diversifying investments to which insurance companies cleaved in Solvency I, and which limited the proportion of the balance sheet invested by issuer category and by issuer, are an embodiment of this. It also corresponds to the popular wisdom that has filtered into the proverb "don't put all your eggs in one basket" in several languages, and not "put all your eggs in a quilted basket" (toward which risk-based but ban-free regulations, such as Solvency II, tend).

4. Conclusion

"[To make] a strenuous effort to put aside some of the artificial schema we interpose unknowingly between reality and us. What is required is that we should break with certain habits of thinking and perceiving that have become natural to us. We must return to the direct perception"

H. Bergson, *La Pensée et le mouvant*, 1938
(Second lecture at the University of Oxford, 1911)

4.1 Tools that damage the quality of decision-making

The economic and financial tools for apprehending risks on the basis of statistical indicators are founded on the conflation of two phenomena of different natures: heterogeneity, for which statistical concepts have emerged and in reference to which we continue to intuit these concepts, and randomness, onto which these concepts have been wrongly transposed.

The pertinence of certain tools is contingent on the observer's position in respect to the phenomenon he seeks to understand. The transposition onto random situations of tools conceived for understanding heterogeneity is exceedingly easy from the standpoint of mathematical formalism, but it is based on the identification of two decision-makers whose positions are radically different, such as an insurer, who *knows* that 1,000 of his 10,000 policyholders will develop a cancer, and a policyholder, who *does not know* whether or not *he* will develop a cancer. For observers in a random situation, this transposition, which leads to the use of statistical indicators,

(i) is not pertinent because it relies on a logical mis-reasoning,
(ii) gives rise to an erroneous mental representation of the situation by creating an illusion of predictability,
(iii) does not provide the means to make the actors aware of their responsibility,
(iv) and does not allow for a quantitative objectivation.

We must insist on the second point here: the danger posed by the use of these tools is not due so much to the unfounded transformation of uncertainty into risk as it is to the fact that they make the randomness of the situation disappear from the point of view of the person using them. Indeed, the essence of risk is a lack of knowledge concerning output; measuring the randomness through one or several deterministic figures, be they volatility, VaR, TVaR, or any other indicator, changes the very nature of the concept: in affirming, for example, that "risk is volatility", we move from a random phenomenon to a deterministic measurement that generates a sense of predictability, an inevitable confusion, and a poor understanding of the decision situation. Hence, the "information" provided is likely to harm the quality of the decision-making.

4.2 An alternative methodology

EU theory, common investment technologies, and quantitative risk management practices are based on the optimization of statistical indicators (expected value, variance, VaR, etc.) and are therefore subject to these failings. In order to avoid these failings, we propose an alternative method for making decisions under uncertainty: to (1) identify the scenario in which we believe and those that we fear – our “Possibles” –, and (2) carry out a maximization of the former under the constraint of the latter being acceptable. The crucial point here is that the actual decision is made during the first step (identification of the Possibles), with the second step (maximization) being only a technical, mechanical step. This methodology of decision-making analysis is based on an underlying empirical description (Solé, op. cit.) and may be linked to the practices of some CROs (Mikes, op. cit.) as well as to certain procedures employed by regulators (e.g., stress test approaches).

On one hand, this methodology improves the pertinence of the decision-making framework, and on the other, it prohibits actors from abdicating responsibility: by nuancing the artificial distinction between analysis and decision, it enables the decision-maker to reason using tangible criteria and situates subjectivity at the right level.

On the operational level, however, it is a time-consuming method, and it can hardly fit into a mathematical optimization schema. Furthermore, it requires two important renunciations: for the decision-maker, who loses the illusion of predictability of an illusory heterogeneous future transformed into an unknown outcome, it eliminates a psychological crutch and it forces him to take responsibility for a decision *under ignorance*: “I don’t know… and I choose an option nevertheless”; for the theorist of decision under uncertainty, it eliminates both the frame of reference that enables the definition of a rationality as well as the hope for a generalized theory, leaving nothing but a methodology in irreducibly contingent contexts.

These costs may seem significant, but in our view, the proposed methodology does not so much improve pertinence as it does *create* pertinence *where there was none*, and where standard practices damaged the understanding of situations.

4.3 Fields of impact to explore

The distinction between randomness and heterogeneity that we have brought to light directly impacts theoretical perspectives (decision theory), societal debates (e.g., the legitimacy of price optimization and rate discrimination in insurance, governmental decision-making), and the steering of financial enterprises (investment decisions, risk management, regulation).

From an operational standpoint, the fruitfulness of this distinction in the field of risk management must be explored further, especially in regard to prudential regulation and investment choices. The distinction can be productive in other fields as well: in the field of financial regulation, for example, it can suggest evolutions in governance. Thus, while activities concerning the calculation of capital requirements are generally attributed to the CRO, due to their claim of capturing risks, we have seen that this calibration rests on a foundation shared with profitability measurement tools and that it is arbitrary from the point of view of risks – as such, this would seem to argue in favor of such responsibilities being steered by the CFO along with the other balance sheet parameters. Conversely, as risk management necessitates the imagination of futures, it would make sense to entrust strategy function to CROs.

From a philosophical perspective, the impasse in which we find ourselves concerning the distinction between measure of randomness and randomness of measurement poses the question of what could be considered a model in a random, non-predictable, and non-back-testable situation.

Moreover, from a methodological point of view, the non-pertinence for decision-making of the decomposition of a heterogeneous phenomenon into random phenomena, which is at the heart of Samuelson's reasoning (op. cit.), can certainly be generalized to other cases of repeated games, and the consequences remain to be properly explored.

The consequences of questioning the boundaries between the rational and the psychological or cognitive (resulting from the criticism of the reference postulated as rational) merits more thorough exploration. Additionally, it appears that, in order to make further progress in normative decision theory, the axes of development switch from the quality of the decision *per se* to the *consciousness* generated by the decision-making mechanism. They thus call for a greater focus on psychology and behavioral economics to better understand what is designated by the term "cognitive bias", both individual and collective, as well as the fields of management, so as to further research into the design of descriptive tools and into specifying how to locate the balance between bias and margin of error.

We should also note that, in light of this contribution, the usual experiments in behavioral economics do not seem to allow for a correct understanding of our behavior in regard to what we commonly call risk. Indeed, for obvious questions relative to financial means, they imply "benign" decisions. These are therefore soluble in heterogeneity: since they have low impact, they can mutualize with other every-day situations (e.g., the randomness linked to the fact that you do not always count coins when you are given change, or the fact that you often don't know if you have overpaid for a bottle of wine until you have drunk it). In this sense, they cannot properly illuminate decisions in truly random situations. In order to successfully gather data, we must either proceed by non-experimental observations or implement protocols in collaboration with those who have the means to do so[20].

Finally, while the renunciation of the ambition to have a decision theory that can be incorporated into microeconomic modeling limits the prospects of social engineering, it does however open the perspective of new worlds, as Solé highlights. Von Mises (op.cit.) also points to this, though negatively, when he reminds us that *"[F]or acting man the future is hidden. If man knew the future, he would not have to choose and would not act. He would be like an automaton, reacting to stimuli without any will of his own."*

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[20] e.g., partnering with producers of game shows, where the stakes are surely high enough, despite the predicament that this would permit us to study random situations in terms of gains but not in terms of losses.

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